Effects of Disparate Information Levels on Bridge Management and Safety

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I, Ciarán Hanley, certify that this thesis is my own work and has not been submitted for another degree at University College Cork or elsewhere.

Ciarán Hanley

Executive Summary

Maintenance planning and life-cycle assessment methods for bridge networks have received large research interest for many years; with modern emphasis often based on probabilistic approaches, due to their ability to handle uncertainty. This allows for risk-based approaches to quantify structural safety, which is largely seen as a superior approach than the deterministic methods typically used in practice. Structural safety can broadly be defined as an acceptable level of chance/probability that the structure will not fail in its function; i.e. to resist the loads/actions to which it is subjected to. The structural reliability method provides for the computation of structural safety by accounting for probabilistically uncertain load models and uncertainties around the return period of extreme load events, as well as the uncertainty in the resistance capacity of the structural system. In addition to a single-point-in-time evaluation of structural safety, the life-cycle performance can be evaluated using physical models for future deterioration; again, constrained under uncertain information about future deterioration. However, being a probabilistic method, it can be somewhat subjective in nature, based on the availability of accurate data and the reliance on expert knowledge, and thus sensitive to the parameters of the input model which rely on the level of information available for the problem at hand.

While structural safety is the apex of modern maintenance planning and life-cycle assessment, the most prevalent performance indicator for which future maintenance and intervention decisions are made come from visual inspection based condition ratings. These visual inspections are used to evaluate the extent of deterioration present and assign a condition rating based on a predefined scale of damage, after which bridge managers trigger further assessment or intervention actions based on acceptable damage levels. Again, in evaluating a single or small number of bridges, there is a degree of subjectivity and reliance on expert knowledge that is also seen with probabilistic assessment methods. Unlike structural reliability, which often suffers from a lack of available or accurate information, condition rating data for large bridge networks generate a large repository of data which provides an excellent opportunity to look at data on a larger scale than is currently implemented in practice. This results in disparate levels of information being available for bridge networks, with large amounts of lower level information and small amounts of detailed information.

In this thesis, how disparate information levels affect these two assessment

methods will be explored and efforts to mitigate against the uncertainty in the information will be suggested. It will be shown that:

- Reliability-based calibrations of bridges are possible through observed clustering of parametric importance and sensitivity measures, based on uncertainty in relation to the available information for probabilistic modelling (Hanley and Pakrashi 2016)
- Existing bridges assessed under code-defined traffic load are sensitive to safety reclassification due to evolving definitions, leading to misinterpretation of the actual state of the structure and, thus, a misallocation of resources (Hanley et al. 2017a)
- Bridges designed under modern, more conservative code-defined traffic load models and assessed under probabilistic load models can expect a longer projected service life before intervention is required, and that the initial construction cost of this conservatism is largely offset when life-cycle cost is considered (Hanley et al. 2016a)
- The use of multivariate analysis methods are applicable to modern bridge management systems that store large amounts of data, and that these methods can provide for clustering of bridges based on their structural forms and states of disrepair (Hanley et al. 2015)
- Large groups of specific bridge types have well-defined, consistent factor structures, whereby a bespoke linear combination of individual elemental condition ratings provide an accurate assessment of the bridge's overall condition rating; improving on currently implemented decision tools in existing bridge management systems (Hanley et al. 2016b, 2017b,c)

This work provides a basis for which further research can be undertaken into developing an information-driven probabilistic decision making framework, leading to the quantification of the value of disparate information levels. The potential future applications include incorporating the derived underlying factor structure of large data-sets of bridges directly into structural reliability methods through probabilistic graphical models, such as Bayesian Belief Networks; thus providing a more robust, information driven framework from which to make decisions under uncertainty.

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"The content is more important than the deadline, really..."

— V. Pakrashi, c2016

Chapter 1

Introduction

1.1 Motivation

Bridges are a crucial aspect of transport infrastructure networks which drive modern economies, linking regions and populations otherwise separated by natural or man-made obstacles. Often times, regional and national transport networks can be dependent on the performance of a small number of bridges to complete the network. When these bridges fail or cease to operate due to safety or serviceability concerns, the economic losses incurred due to the restriction of free movement of goods and services are severe. In a report by the American Society of Civil Engineers (ASCE 2011), it was estimated that the cost of a deficient surface transport network, due to deteriorating conditions, to homes and businesses was $130 billion; while failure to halt this deficiency would cost the American economy 400,000 jobs by 2040. Due to these consequences, stakeholders of bridge networks are tasked with responsibility of ensuring the safe, reliable, and functional performance of bridges within their purview. In order to meet this task, bridge managers must look to innovative methods to guide their decision making process with regard to intervention policies of network assets in the latter stages of their service life.

A review of the national bridge stock in six European countries showed that the majority of bridges were built in the post-war period of 1945–1965 (Žnidarič et al. 2011), while in the United States, the average age of the national bridge stock is 42 years; 11% of which is said to be structurally deficient and 25% said to be "functionally obsolete" (ASCE 2013). However, it is at this present time, when a significant portion of the bridge stock demands resources, that bridge managers find themselves having to operate under significant budget restraints due to recent economic conditions. In this regard, the most effective allocation of resources becomes a primary motivator for practice and research, with risk-based methods emerging as a prominent solution to effective infrastructure asset management under uncertainty.

Risk-based management under uncertainty is the state-of-the-art in terms of life-cycle prediction of structure and infrastructure systems; whereby probabilistic methods form the underpinning concepts for design and assessment, and are consequently the main performance indicators of a structure (Ellingwood and Frangopol 2016, Biondini and Frangopol 2016, Ghosn et al. 2016a,b, Sánchez-Silva et al. 2016, Lounis and McAllister 2016). Structural reliability methods are a tool used to incorporate uncertainty into performance assessment of structures, by treating them as probabilistic systems that can vary with time as opposed to deterministic in nature. In its most elementary form, structural reliability analysis is conducted under the context of the equation:

$$g = R - S \tag{1.1}$$

Where g is a limit-state; R is a resistance variable; and S is a load variable. In a general sense, R is a resource variable and S is a demand variable (Lemaire 2009). In this basic example of a simple structure, the limit-state is violated if an applied load S exceeds the structures capacity R it resist it; and thus $g \leq 0$ is the failure domain of the structure. For common structures and structural systems with multiples of variables, the solution of this equation requires transformation to the probabilistic space, such as through a Rosenblatt transformation (Rosenblatt 1952), or simulation methods, such as Monte Carlo simulation; and can often yield different solutions based on small changes in the model parameters. Due to this sensitivity, structural reliability assessments of existing structures are typically more bespoke than conventional, widely used assessment practices, and are thus less generic and uniform. While guidelines exist for structural reliability modelling, the model parameters and variables are often chosen based on existing best practice as opposed to site-specific information on the load effects and material properties present in the system. This information can be obtained by employing modern weigh-in-motion (WIM) technologies for evaluation of local traffic effects, or through destructive and non-destructive evaluation (NDE) methods to establish local material properties and environmental effects. Both methods, however, require considerable resources, and so this information is not typically available on a wider scale.

However, the most ubiquitous source of information on the current state of bridges is condition rating data obtained through visual inspection. Visual inspection is the most common form of bridge assessment, as it is inexpensive relative to the other methods in terms of budget allocation and required time. These inspections are used to classify the operating states of bridges, as well as highlight any significant damage which may have occurred since the previous inspection or last intervention. It is typically not used as a damage prediction tool, in that it cannot inform the inspector of deterioration rates of the structural materials, yet it can be used in model updating based on the expected condition state due to *a priori* analysis. In this regard, it can be a useful tool not just for classification of operating states, but in the advancement of more sophisticated assessment techniques. Further to this, there is a degree of subjectivity within the condition assessment of bridges through visual inspection, which results in somewhat ambiguous classifications of condition states. If used to update probabilistic models, this ambiguity must be mitigated in order to ensure an appropriate classification of safety, and consequent resource allocation. By investigating the available information at a larger scale, it should be possible to reduce the ambiguity in this assessment method by employing multivariate analysis techniques in order to gain a deeper understanding of the appropriation of condition ratings, with an effort to establishing a more objective framework from which to feed into further levels of assessment.

As bridge maintenance management evolves over time, we now stand at a juncture where significant data collection is being carried out at a network level, but different levels of information are available at different amounts and with different levels of accuracy or uncertainty. There is no commentary at this moment on what such variations of information at different levels lead to and how it can inform future decision making for bridges; especially at a network level.

1.2 Background

Historic, current, and predicted information of individual bridges, or a collection of bridges, in a network have increased significantly in several networks globally. A focus on maintenance and management of bridge networks to a minimum acceptable level of performance, usually in the presence of funding constraints, remains a challenge to bridge managers. Bridge management systems provide a framework from which to implement intervention strategies, in which a large amount of data is collected and archived for these networks. This data can be sourced from historical records, inspection records, testing results, and deterministic and probabilistic assessments. Advancement in digital technology is expected to lead to a more intensive data-driven decision making platform for networks of bridges and individual bridges. However, there is currently an imbalance in the quantity and quality of information available from each data source. Condition rating data from visual inspection is available to bridge managers of networks in large quantities, but contains very limited to no information around the structural capacity of a bridge. Contrarily, structural reliability methods, considering site-specific traffic loading and material strength with degradation models, provides a greater understanding of structural capacity; but data from these methods are not as widely available. While studies in relation to individual tests, inspection methods, or assessment techniques are well-established, very few studies exist comparing the effect of information at different levels on the assessment of individual bridges and of a network. A better understanding of how information at different levels can impact bridge assessment can lead to better management of these networks. Patterns may develop when this information is available for a set of bridges in a network, and this can lead to the development of network-based calibrations of their performance by estimating these patterns as a performance signature of the network. The studies can also be indicative of what types of bridges tend to cluster together and how degradation processes and other decisions on loading eventually affect bridges. A greater understanding of information at different levels can lead to better managed bridge networks and a more judiciously chosen monitoring and intervention options.

Early works by Liu and Frangopol (2004) and Estes and Frangopol (2003) have investigated possible presence or absence of correlations for bridges, but there has not been sufficient real data to develop a network calibration. When estimates of degradation are present, it is possible to assess and optimise costs related to rehabilitation in a life-cycle format (Kong and Frangopol 2003). The most direct study to-date linking different levels of information relates to 14 bridges, within relatively close geographical proximity to each other, in a small urban network in the United States; which led to the development of some amount of correlation between these levels of assessment, despite the small sample size and some significant variation (Akgül and Frangopol 2004a). More simplified models have been used in larger networks for assessing the effects on service life for specification changes (Kirkpatrick et al. 2002), which provides incentive for investigating how other such changes in standard specifications might affect bridges over time. Correlations from a range of bridges are more well-researched in terms of testing, where comparative performance in relation to corrosion (Pakrashi et al. 2012a) and strength (Breysse 2012) are often carried out; usually as part of a special inspection (Harries 2009). While performance updating (Akgül and Frangopol 2003, Maes 2002) and cost-effective repair optimisation (Orcesi and Frangopol 2011, Stewart et al. 2004) naturally come from many such assessments, it is observed from the literature that the availability of real data or actual policy changes provide a better insight to the real condition of a network; and thus the methodologies are better compared for such realistic considerations. This is not counter-intuitive, but the availability or consideration of such data for the same and similar networks are still not readily available and there is a necessity of presenting more such investigations.

Limited information and variations in uncertainty exist in every level of bridge assessment at a network level, and understanding the effects of such variation better, using existing networks and representative bridges, is important; especially when instrumentation and testing of bridges is becoming more commonplace due to technological advancement. However, most networks still rely heavily on visual inspections for maintenance planning, as it is the least resource intensive method of obtaining some information regarding the state of a network; despite the obvious limitations associated with such assessments. Nevertheless, when this information is available at a network level, some patterns and inferences may be drawn regarding the state of the network and its evolution due to degradation over time (Reale and O'Connor 2012, Lovejoy 2003). Information obtained in this manner can be linked to the Value of Information format, typically using Bayesian Belief Network modelling (Kallen 2007), and eventually leading to sensor placement strategies (Malings and Pozzi 2016). While these advantages are present, the variations or uncertainties around this information is also observed to be an important aspect for bridge management (Moore et al. 2001, Figueiredo et al. 2013, Frangopol and Bocchini 2012). While most managed networks seem to align to minimum levels of safety and serviceability limits, a significantly large number of bridges on non-national or secondary roads seem to be structurally deficient (Chase and Laman 2000). Under such circumstances, the margin of safety is often less than what is typically obtained for better managed networks, and probabilistic assessments of such bridges can lead to lower reliability indices than usually acceptable (Akgül and Frangopol 2003, Leander et al. 2015).

For probabilistic assessments of real networks, there is inadequate information around how parameter importance measures perform and if similar types of bridges tend to cluster in values or not. The variations around such analyses are primarily due to uncertainty in capacity variables; yet the impact of load variables, especially for traffic loading, due to changes of regulations over time on bridge assessment can provide significant variation in assessment outcomes. This leads to a circumstance where more onerous classifications of safety occur in certain periods of time; since the assessments typically were and currently are carried out using deterministic methods with prescribed loading. For inspection data, there are questions around how to reduce and cluster such data when available from a network, and can this data be used to provide insight to the condition of the network; leading to a calibrated signature of the network. Impacts of disparate levels of information on bridge maintenance and management is thus an active and important field, especially when more information and data is becoming available. While the connection between different levels of information may still not have enough available data to draw a definitive conclusion, understanding the influence of variation of information at different levels for a bridge network leads to a first step towards future assessment of these networks in a data-driven environment. This thesis attempts to address this issue using as much real data, information, and policies as possible.

1.3 Research Objectives

The objective of this thesis is to investigate the effect of disparate information levels on bridge management and safety, and how the levels of information available to the engineer can severely impact the stability of probabilistic assessment. With the effective use of available information, and the understanding on the limits of this information, it should be possible to further the application of probabilistic assessment methods in the evaluation of existing bridges and understand the entire process of bridge maintenance management on a deeper level.

The specific objectives of the thesis are to:

1. Demonstrate the effect of uncertainty on reliability assessment of bridges

by studying the sensitivity of the model parameters and their influence on the model solutions

2. Investigate the evolving nature of codes for bridge design and assessment, and show how variations in traffic loading definitions can introduce a level of uncertainty to a network assessment, independent of the deterioration of the bridges
3. Evaluate the effect on life-cycle costs for bridges designed in different eras and, thus, under different traffic loading definitions; with a view to determining how conservatism at the design stage can reduce the requirement for financial investment in the latter stages of a bridges service life
4. Use multivariate analysis techniques on condition rating data within bridge management systems to develop an improved understanding of the behaviour of specific bridge types in the context of visual inspection data
5. Relate the derived models back to risk-based evaluation and explore the potential applications in which the use of multivariate techniques on existing data can be used

1.4 Scope of Work

The scope of the work is mostly limited to the evaluation of short span bridges (Figure 1.1), as these bridges form the majority of national networks within Europe, subjected to traffic loading only. The bridges assessed probabilistically are reinforced and prestressed concrete bridges; being the predominant type of modern bridge in Ireland. The assessments incorporates the use of a statistically defined traffic load model, from which comparisons will be made to the traffic load models defined in codes used in Ireland in the past and present. As the load model used defines the annual maximum value of traffic loading, the reliability indices determined using this model are considered to be annual reliability indices and nominal values; from which it is possible to provide comparisons. The data used for the corrosion modelling has been based on results obtained largely from studies conducted on bridges in the United States, and so the results presented herein are non-specific to Irish bridges and corrosion rates. The time-dependent reliability assessment will be based on a single-point-in-time assessment of the future reliability of the bridge at a specified time, assuming

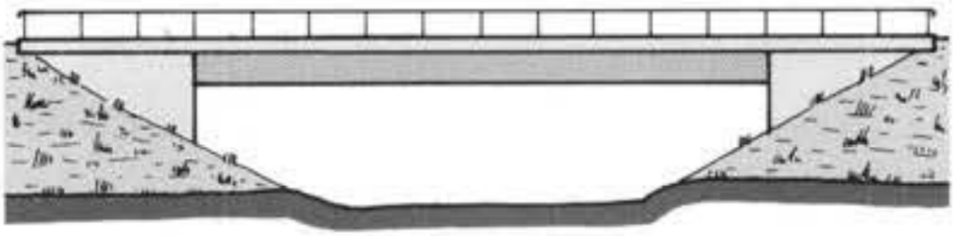

Figure 1.1: Typical arrangement of short, single span highway bridge (Leonhardt 1982)

failure has not already occurred. The level of uncertainty in relation to the flexural capacity of these bridges is investigated, from which it can be seen that the results can be quite sensitive to the levels of information available to the engineer. In this regard, the most ubiquitous information source in bridge management, condition rating data, is used from which to establish a reliable model from which to drive a greater level of information into structural reliability modelling. The primary source of data used in this analysis is the condition rating data of single span masonry arch and reinforced concrete bridges, being the most numerous type of bridge typically seen in existing stock in Europe. These bridges are not identified in the thesis, but are defined by their parameters and their presence within respective data-sets. A targeted literature review is included at the beginning of Chapters 3-6.

1.5 Outline of Thesis

This thesis is organised into seven chapters, where:

- **Chapter 1** provides an introduction to the topic and defines the scope and objectives of the presented research
- **Chapter 2** gives the theoretical background to the probabilistic method of structural reliability analysis, and is presented in the context of its sensitivity to sources of information used in the construction of a probabilistic assessment model
- **Chapter 3** considers model uncertainty for reliability analysis, and proceeds to estimate the level to which the computed reliability indices β are a function of the level of uncertainty in the model parameters. Additionally, useful by-products of reliability analysis, in the form of sensitivity

and parameter importance measures, are shown to give guidance on the level of uncertainty in the model and what parameters to target to reduce uncertainty through information gathering

- **Chapter 4** models the effects of changes in traffic loading definitions over time and the fact that such definitions may influence the computed level of safety of bridges. These estimates identify periods of time when such definitions have had the maximum effect. Further to this, it is shown that bridges designed under modern, conservative design standards might enjoy a prolonged life-cycle and that the increased cost associated with more conservative design can be significantly offset by life-cycle cost reductions seen due to the reduced necessity for essential or preventative maintenance
- **Chapter 5** investigates the concept of multivariate data techniques and their application to condition rating data in bridge management systems. Specifically, multivariate data reduction techniques will be explored in an effort to reconstitute the large data set into a reduced space that explains the most variance in the data-set
- **Chapter 6** shows the applicability of using multivariate data techniques on existing available information, and how this can be used to derive an improved model for network performance through the use of latent variables and underlying factor structures of bridge management systems, by utilising perceived patterns in condition rating signatures for different bridge types in varying locations
- **Chapter 7** gives a summary of the research contributions contained in this thesis, including a critical appraisal of the work and its limitations; as well as suggested directions for future work derived from this research

1.6 Research Output

The following publications represent the primary dissemination of the research contained in this thesis, to date.

Book Chapters

- Pakrashi, V. & **Hanley, C.** (2015). Performance-based design of structures and methodology for performance reliability evaluation. In *Structure Design and Degradation Mechanisms in Coastal Environments*, Chapter 6, pp 247–284. ISTE Ltd., London, UK.

Journal Papers

- **Hanley, C.**, Matos, J.C., Kelliher, D., & Pakrashi, V. (2017). Integrating multivariate techniques in bridge management systems. *Journal of Structural Integrity and Maintenance*, pp 1–22 (In-Press).
- **Hanley, C.**, Frangopol, D.M., Kelliher, D., & Pakrashi, V. (2017). Reliability index and parameter importance measures considering changes in bridge traffic loading definitions. *Proceedings of the ICE - Bridge Engineering*, pp 1–23 (In-Press).
- **Hanley, C.**, Matos, J.C., Morais, J.G., Gudmundsson, G., Lapthorne, J., Kelliher, D., & Pakrashi, V. (2017). Latent variable approach to condition assessment of bridges through multivariate data reduction techniques. *Journal of Bridge Engineering*, pp 1–20 (In-Revision).
- **Hanley, C.** & Pakrashi, V. (2016). Reliability index analysis of a bridge network in Ireland. *Proceedings of the ICE - Bridge Engineering*, 169(1):3–12.
- **Hanley, C.**, Kelliher, D., & Pakrashi, V. (2015). Principal component analysis for condition monitoring of a network of bridge structures. *Journal of Physics*, 628:012060.

Conference Papers

- **Hanley, C.**, Matos, J.C., & Pakrashi, V. (2016). Principal component analysis as a comparative technique for bridge management systems. In *Bridge Performance Goals and Quality Control Plans - 3rd Workshop of COST Action TU1406*, Delft, Netherlands.

- **Hanley, C.**, Matos, J.C., Kelliher, D., & Pakrashi, V. (2016). Integrating multivariate techniques in bridge management systems for life-cycle prediction. In *Civil Engineering Research in Ireland (CERI2016)*, pp 237–242, Galway, Ireland.
- **Hanley, C.**, Frangopol, D.M., Kelliher, D., & Pakrashi, V. (2016). Effects of increasing design traffic load on performance and life-cycle cost of bridges. In *Maintenance, Monitoring, Safety, Risk and Resilience of Bridges and Bridge Networks (IABMAS2016)*, pp 222–229, Foz do Iguaçu, Brazil.
- **Hanley, C.** & Pakrashi, V. (2014). Parameter importance measure studies on prestressed concrete bridges in Ireland. In *Civil Engineering Research in Ireland (CERI2014)*, pp 51–56, Belfast, Northern Ireland.

Chapter 2

Background to Probabilistic Assessment

2.1 Introduction

As infrastructure deteriorates over the course of its life-cycle, it is important to ensure the continued safety and serviceability of the various elements. To achieve this, it is necessary to be able to predict the point in time at which an infrastructural asset will deteriorate to a critical condition state. To predict this point in time, it is necessary to consider how this deterioration will affect the structural capacity of the asset, as well as any future increase in the applied loading; all of which have an associated level of uncertainty, hindering an accurate prediction (Ang and Tang 2007). Thus, advanced methods are required that allow for the consideration of all pertinent information in an analytical assessment model. There exists a hierarchy (Figure 2.1) for the levels of assessment associated with an infrastructure network (Pakrashi et al. 2012b), for which all levels are included into the decision making tool. The most advanced level in this hierarchy is probabilistic/reliability methods, due to its role in the establishment of key performance indicators for structures.

Reliability is regularly related to the probability that failure will not occur; thus, it is complementary to the failure probability P_f and, consequently, this leads to the estimation of probability of structural safety over a prescribed period of time. Often, this prescribed period of time is the 'design life' of a structure; typically agreed to be 120 years (CEN 2002). The use of probabilistic methods allow for the treatment of structures as probabilistic systems rather than determinis-

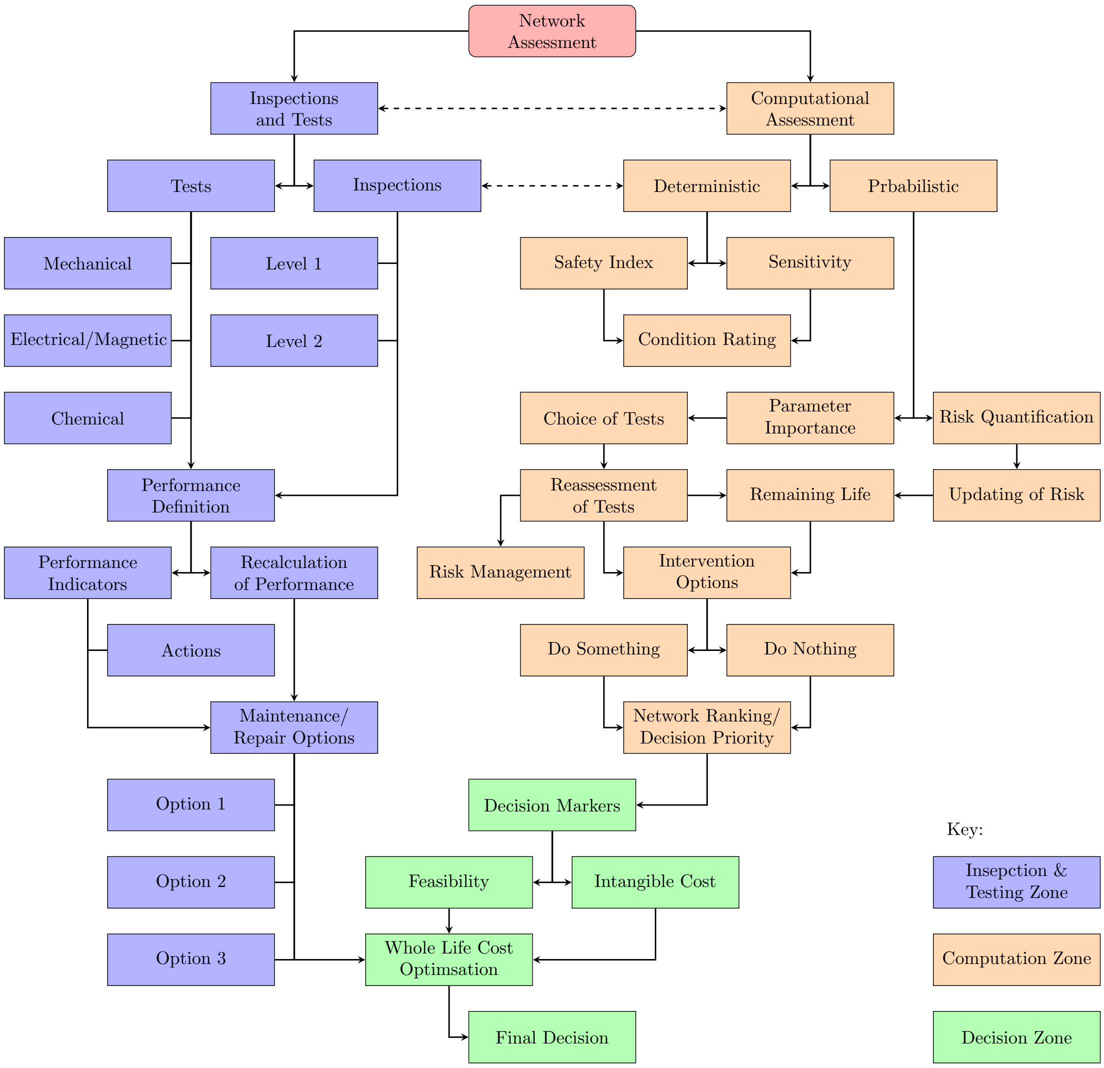

Figure 2.1: Infrastructure assessment hierarchy, adapted from Pakrashi and Hanley (2015)

tic, a treatment which is more accurate to the actual realization of an engineering structure. Reliability methods provide for the computation of safety based on varying load models and uncertainties around the return period of extreme load events, in addition to addressing the uncertainty in the resistance capacity of a structure. Previously, the wide-spread application of structural reliability analysis was hindered by high computational demand, but modern advances in computing technology have overcome these issues (Ellingwood 2006). Thus, there remains no effective restriction on the extensive implementation of the method for infrastructure assessment or design.

Probabilistic methods can sometimes be subjective in nature or may be based on engineering judgement to a certain extent, as the input variables required for the analysis can only be modelled based on the level of information available about the problem at hand. From this, the lack of information, or uncertainty in information, becomes a significant consideration in reliability analysis. Uncertainties can be simply defined in two ways: aleatory and epistemic (Faber 2005). Aleatory uncertainty relates to uncertainties that are inherent to the problem and must be accounted for, whereas epistemic uncertainties are those which can be reduced or mitigated through the collection of information about the problem, and the refinement of the model used in the analysis.

A brief overview of the structural reliability method is presented in the following sections for completeness, with the presented theory being the basis for further discussion in later chapters.

2.2 Code Treatment of Structural Reliability

2.2.1 Formulation of Reliability Analysis

The basic formulation of structural reliability can be expressed through a model of a single load effect variable *S* resisted by a single resistance variable *R*, with each having its own probability density function. In the general case for engineering structures, the limit state is violated if *S* exceeds the value of *R*. This limit-state can be expressed in the following form:

$$g = R - S \tag{2.1}$$

This can also be described in different but probabilistically equivalent limit-state functions, depending on the criteria of the problem being assessed:

$$g = 1 - \frac{S}{R}, \qquad g = \ln\left(\frac{R}{S}\right) \tag{2.2}$$

The probability of this violation is identical to the probability of failure P_f. This concept has its most basic form when considering a single structural element for an ultimate or serviceability limit-state, which is denoted by the limit-state function, $G(\mathbf{X})$.

$$P_f = P(R - S \leq 0) = P[G(R, S) \leq 0] = P[G(\mathbf{X}) \leq 0] \tag{2.3}$$

where P represents probability. In the case that the basic variables in the limit-state function are independent:

$$P_f = \int_{R<S}\int f_R(r)f_S(s)\mathrm{d}r\mathrm{d}s = \int_0^{\infty}\int_0^{S} f_R(r)f_S(s)\mathrm{d}r\mathrm{d}s = \int_0^{\infty} F_R(s)f_S(s)\mathrm{d}s \tag{2.4}$$

where $F_R(S)$ is the cumulative distribution function, with the convention being used of upper case letters denoting the cumulative distribution of its lower case counterpart. The simplest form of structural reliability analysis can be evaluated by modelling each random variable in the problem as a normal distribution and using only the first two moments: mean μ and standard deviation σ. The reliability index β and failure probability P_f can then be approximated as (Cornell 1969):

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = -\Phi^{-1}(P_f) \tag{2.5}$$

However, the output from this method must strictly be considered nominal and is useful only when employed on a comparable basis to other analyses of the same type. Additionally, the use of this method assumes a linear limit-state function with a normal distribution. However, limit-states are typically observed to be nonlinear in practice, which make it difficult to obtain the first two moments of the limit-state function.

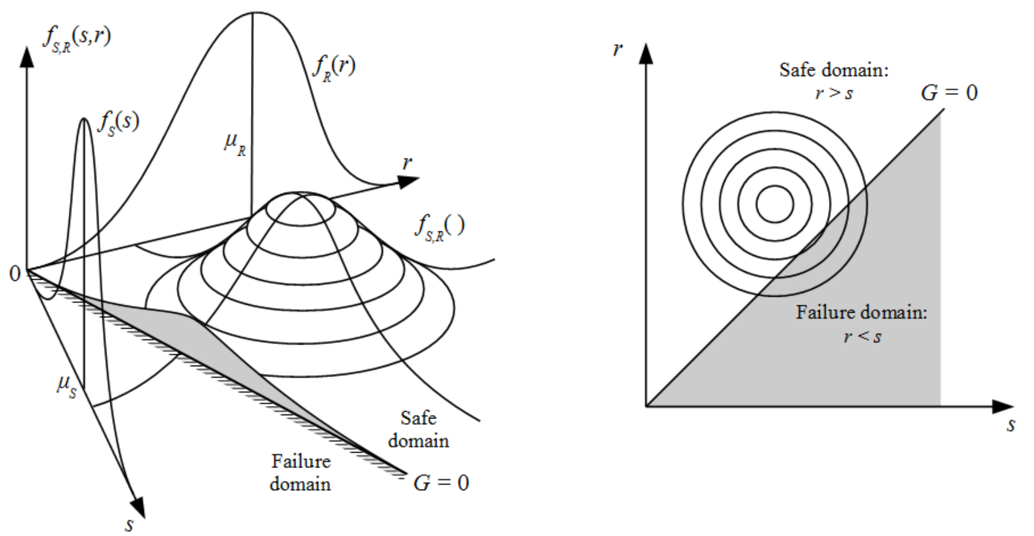

Figure 2.2: Conceptualization of the reliability problem (Melchers 1999)

When conducting a structural reliability analysis, four levels have been defined with increasing levels of complexity. Level I, the most basic level, is the commonly seen partial factor approach used in design codes where deterministic values are used and uncertainty is accounted for by way of partial factors. Level II methods incorporate the idea of model uncertainty by describing the input parameters as normal distributions with mean and standard deviation values. The use of this method yields a nominal failure probability useful for comparison purposes. Level III methods involve establishing the failure probability using advanced methods, such as transformations and simulations, and can be referred to as “exact” methods. Level IV methods expand on the “exact” methods by incorporating economic models in order to generate a risk analysis.

2.2.2 Incorporation of Reliability Analysis into Normative Documents

The role of normative documents, such as structural design codes, is generally to ensure a degree a harmonisation of structural design, in an effort to provide minimum safety levels of structures. Having employed the use and followed the rules of a design code, the designer should have reasonable confidence in the safety and serviceability of the designed structure (Melchers 1999). To this end, a design code needs to be based on advanced methods for predicting structural safety, but also to be accessible enough to the end-user/designer. To address this issue, modern structural design codes (DNV 1992, CEN 2002, JCSS 2000) implement the concept of partial factors γ for both loads and resistance variables. The partial factor method, also called the load-resistance factor reduction (LRFD) method, is designed to be able to apply probabilistic uncertainty to deterministic design variables (Ellingwood 1996). For the reliability method, an applied load above the mean value μ_S is said to be unfavourable, while a resistance value below the mean μ_R is also deemed unfavourable. Thus, the partial factors are applied to simulate such an unfavourable scenario. The values for these partial factors can be determined through calibration to the limit-states equations specified in the code against the target reliabilities established to ensure adequate safety is provided.

2.2.3 Reliability Targets

The reliability target is the level for which a structure should perform at a minimum. In design, it can be used as the minimum target benchmark to ensure that certain safety levels are achieved. Rather than simply be based on a pass-fail approach for all structures, the target reliability can be optimized to different scenarios where failure consequences may be considered on different levels, and where the nature of the structural failure can be predicted. Thus, target reliability indices can be seen to be situational. The specification of target reliabilities has been included in numerous design standards prepared by *Det Norske Veritas* (Table 2.1), the *European Committee for Standardization* (Table 2.2), and the *Joint Committee on Structural Safety* (Table 2.3).

Table 2.1: Target reliability for failure types and consequences (DNV 1992)

	Consequence of failure	
Class of failure	Less serious	Serious
I – Redundant structure	3.09	3.71
II – Significant warning before the occurrence of failure in a non-redundant structure	3.71	4.26
III – No warning before the occurrence of failure in a non-redundant structure	4.26	4.75

Table 2.2: Target reliability for ultimate limit state (CEN 2002)

	Reference period	
Consequence classes	1 year	50 years
Low consequence for loss of human life, and economic, social or environmental consequences small or negligible	4.2	3.3
Medium consequence for loss of human life, economic, social or environmental consequences considerable	4.7	3.8
High consequence for loss of human life, or economic, social or environmental consequences very great	5.2	4.3

A reference period refers to the period of time used as a basis for assessing stochastic actions. It should be noted that when designing a new structure or assessing an existing structure to fulfil target reliability levels, the uncertainties within the model used will affect the reliability level. Thus, the true relationship between the calculated reliability index and the target reliability index is a

Table 2.3: Target reliability for ultimate limit state at one year reference period (JCSS 2000)

	Relative cost of safety measure		
Consequence classes	Large	Normal	Small
Minor – risk to life is small and negligible economic consequences	3.1	3.7	4.2
Moderate – risk to life is medium and economic consequences are considerable	3.3	4.2	4.4
Large – risk to life is large and economic consequences are significant	3.7	4.4	4.7

function of the commonality of the uncertainties and assumptions used in the establishing models.

2.2.4 Consistency with Deterministic and Semi-Deterministic Methods

As mentioned earlier, the target reliability can be used in order to calibrate partial factors in a design code that used the Load and Resistance Factor Design (LRFD) method, or limit-state design. The typical limit-state equation used in modern design codes is of the form:

$$\phi R_n \geq \sum_{k=1}^{i} \gamma_k S_{km} \tag{2.6}$$

Where the characteristic resistance values are typically reduced by a partial factor ϕ and the characteristic load actions are typically increased by the application of the partial factor γ. As the values of partial factors are derived from probabilistic methods in reference to target reliabilities, codes of practice which use partial factors are said to be probability based codes or semi-deterministic/probabilistic (Vrouwenvelder 1997). These partial factors are derived using the general procedure outlined by Melchers (1999).

2.3 Second Moment Transformation and Simulation Methods

In order to facilitate the broad application of the reliability method, a generalized reliability problem is required to be defined, derived from the load-resistance case presented earlier. However, in most engineering applications, R and S will not comprise single variables but will be a function of a number of basic variables which contribute the limit-state function. All basic variables can be represented by the vector **X**. Now, by expressing the generalized limit-state function as $G(\mathbf{X})$, the failure probability for the joint probability density function $f_X(x)$ can be expressed as:

$$P_f = P[G(\mathbf{X}) \leq 0] = \int \dots \int_{G(\mathbf{X}) \leq 0} f_{\mathbf{X}}(x)\mathrm{d}x \tag{2.7}$$

In most cases of evaluating the generalized failure probability, the integration of the probability density functions cannot be performed analytically, and must be approximated using appropriate methods; of which the two leading approaches are transformation methods and simulation methods. Using simulation methods, such as Monte Carlo methods, the multi-dimensional integral can be evaluated. Conversely, transformation methods are used when bypassing the integration is desirable, and the joint probability density function is transformed to a multi-normal probability density function which can be described by its moments. Although often seen to be competing methods, the belief held by some researchers is that these methods should be seen as complementary; as one method may be more appropriate for a specific problem over another (Bjerager 1990).

It is possible to evaluate the failure probability through direct integration, but only in a limited number of instances; specifically where the limit-state function is linear and all random variables are normally or lognormally distributed. For this reason, it is largely considered an impractical method to solve for the failure probability.

2.3.1 Problem Formulation

The First-Order Second Moment (FOSM) method was developed to linearise the nonlinear limit-state function using a Taylor series expansion about a linearisation point. The location of this point is best chosen to be the design point, being the point of maximum likelihood. However, it has previously been linearised at the mean values of the random variables, giving rise to the name Mean Value First-Order Second Moment (MVFOSM) method. Moreover, the benefit of this method is that it is easier to locate this point than that of the design point, but does not offer as good an approximation. However, linearising the surface at the mean leads to an invariance problem, where the analysis of equivalent limit-state functions will result in a disagreement of the reliability indices. To correct this invariance problem, the first-order reliability method was developed.

2.3.2 First-Order Reliability Method

First-order reliability methods (FORM) involve transforming non-Normal random variables into comparable Normal random variables that can be described using their first-order moments. This can be achieved using methods such as the Rosenblatt (Rosenblatt 1952) or the approximate Nataf transformations. However, by transforming the random variables, the limit-state function is also transformed, and is usually now represented as a nonlinear function. In order to compute the reliability index, FORM requires a linearisation of the limit-state surface at a point that provides a better approximation than seen with MVFOSM. The linearisation is achieved through a Taylor series expansion about a point on the limit-state surface, optimally chosen to be the design point $\mathbf{u}^*$. The prominent computational demand of FORM is through the location of $\mathbf{u}^*$, and methods to locate this point are discussed later. A general algorithm is developed based on the location methods, which is repeated until the solution converges to a point where $\mathbf{u}^*$ and β stabilise in terms of value.

FORM addresses the invariance problem present using MVFOSM by approximating the limit-state surface at a point as opposed to the mean value of the random variables. But, as can be seen, using an expansion method to linearise the limit-state surface becomes less accurate as the level of curvature of this surface increases, and as such, use of FORM becomes less desirable in these scenarios, in comparison to the use of second-order methods.

2.3.3 Second-Order Reliability Method

Second-order reliability method (SORM) is an extension of FORM, but without the need to linearise the limit-state surface. Instead, a hyperparabolic surface is fitted the limit-state surface at the location of the design point. Due to this, SORM is capable of dealing with problems of a higher degree of complexity than FORM, as the method can be extended to highly curved limit-state surfaces.

A number of methods have been proposed to evaluate the failure probability using SORM (Der Kiureghian et al. 1987, Hohenbichler and Rackwitz 1988, Tvedt 1990, Der Kiureghian and Stefano 1991), but the simplest implementation of the method involved asymptotic approximations (Breitung 1984) and multiplied the FORM result by a correction factor:

$$P_f \approx \Phi(-\beta) \prod_{i=1}^{n-1} \frac{1}{\sqrt{1+\beta k_i}} \tag{2.8}$$

As can be seen, the correction factor is a function of the limit-state curvatures k_i at the design point. Thus, the problem reduces to one of determining the curvatures of the limit-state surface.

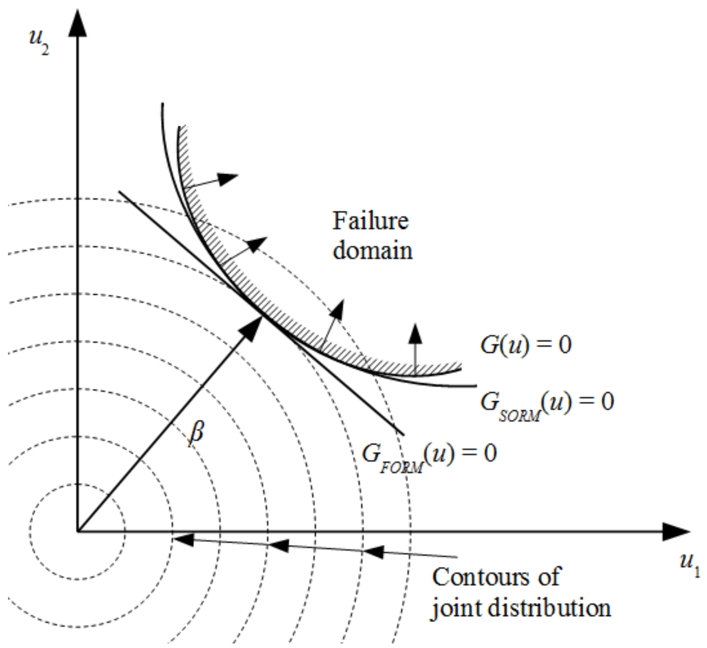

Figure 2.3: FORM linearisation and SORM approximation in standard normal space

2.3.4 Monte Carlo Simulation for Reliability Analysis

A direct method to evaluate the probability density integral for the limit state function is through simulation methods. The use of simulation methods was proposed as an alternative to the high computational demand required for solving through direct integration, and was specifically helpful in solving for non-linear limit-state functions. Monte Carlo simulation involves artificially running a large number of experiments based on the numerical model, with the output being a function of the number of experimental failures observed. When applied to structural reliability, the failure probability is calculated as being the number of the instances the limit-state function was violated across the total number of experiments run *N*.

$$P_f \approx \frac{n[G(x_i) \leq 0]}{N} \tag{2.9}$$

To evaluate the accuracy of a Monte Carlo simulation, the coefficient of variance of the failure probability δ_{pf} should be checked, and is defined as:

$$\delta_{pf} = \frac{\sigma_{pf}}{\mu_{pf}} = \sqrt{\frac{1 - p_f}{N p_f}} \tag{2.10}$$

Values of 2–5% for δ_{pf} are typically deemed to be acceptable. Knowing the acceptable levels of CoV, and having a target reliability index/failure probability, the number of samples required for an acceptable Monte Carlo simulation can be found from the following formula:

$$N = \frac{1}{\delta_{pf}^2} \left(\frac{1 - p_f}{p_f} \right) \tag{2.11}$$

It should be noted that the Monte Carlo simulation method is only a practical alternative method when the number of simulations is less than the number of integration points required for a numerical integration. Additionally, the Monte Carlo simulation can be optimized by sampling in the area of the design point. This greatly improves the efficiency of the method and is referred to as Importance Sampling. However, a transformation method, such as FORM, must be conducted in order to locate the design point.

2.3.5 Practical Implementation Aspects

As the computational difficulties from the past have been rectified through development of robust methods and commercially available software applications which implement them, there remains little reason to hinder the widespread application of the structural reliability method. Remaining difficulties include the sufficient education of engineers to be able to fully utilize the method and work with the output. However, as with any advanced numerical method and associated software application, the quality of the input data is of primary importance as small input errors can manifest in large output errors. To effectively model the input variables, the most appropriate probability distribution must be chosen, of which will be highlighted in the following sections.

2.4 Resistance Modelling Considering Deterioration and Uncertainty

It is evident that the purpose of inspecting a structure is to assess whether it is continuing to perform for its intended purpose. For engineering structures, this is typically borne by whether the structure has the capacity or resistance to sustain the applied load in a safe manner. For new structures, this is simply accounted for in adherence to modern design codes, which have a founding in probabilistic methods. However, for existing structures, the prediction of the actual strength of the structure is often determined using imprecise methods. A condition rating is usually assigned to the structure based on the results of the assessment. For many infrastructure networks, these condition ratings are assigned based on the results of a visual inspection alone. For infrastructure managers to make maintenance decisions for a vast network, while operating under budget constraints, this is often seen as an economical approach. However, it must be noted that visual inspections cannot offer information on how a structure is likely to deteriorate over time (Frangopol et al. 2001) and, therefore, they do not make for a good decision tool when considering future intervention plans. Additionally, visual inspection cannot highlight how a material is performing internally. Thus, in order to effectively allocate future resources for areas of future need, it is necessary to make decisions based on methods that allow the accurate modelling of deterioration.

2.4.1 Uncertainty Modelling

A benefit of using probabilistic methods in the design and assessment of engineering structures is the ability to adequately account for uncertainty in problems. The types of uncertainties typically associated with an engineering problem are (DNV 1992):

- physical uncertainty, which is intrinsically associated with a variable due to naturally occurring randomness in its composition. Efforts to reduce this type of uncertainty can only work to an extent, but it can never be truly eliminated
- the uncertainty related to the measurement and collection of data. This type of uncertainty is usually confined to human error and equipment error, which can be satisfactorily eliminated by calibration to a known state
- statistical uncertainty, which can arise due to the sample size of information used in an analysis, or an insufficient number of iterations used on a convergence
- model uncertainty, which is associated with the construction of the analytical solution. This type of uncertainty, in addition to being a function of other types of uncertainty, is based on decisions made by the engineer, and can increase due to excessive and incorrect simplifications/assumptions made in devising the computational or physical model. Methods to account for this type of uncertainty have been developed to be integrated into the computational model (NKB 1978, O'Brien et al. 2015a)

Another source of error in the determination of a computational model is the selection of a probability distribution for the random variables in the problem. It has been shown that curve fitting a distribution to a set of data will often allow the engineer to select a number of seemingly appropriate distributions that possess a similar form or curvature. However, the tails of these distributions are often vastly different and result in widely varying approximations of failure probability. This usually referred to as the "tail sensitivity problem", and efforts have been made to reduce this effect by standardising appropriate probability distributions for load and resistance random variables (Tables 2.4 and 2.5).

Table 2.4: Material property distribution models (O'Brien et al. 2015a)

Variable	Unit	Distribution	σ	CoV
Modulus of elasticity	N/mm^2	Normal	-	-
Steel strength (reinforcing steel)	N/mm^2	Lognormal	25	-
Concrete strength	N/mm^2	Lognormal	-	0.12–0.22
Area of steel	mm^2	Normal	-	-
Effective depth	mm	Lognormal	-	0.05-0.20
Cover to reinforcement	mm	Lognormal	-	-
Steel strength (prestressing steel)	N/mm^2	Lognormal	-	0.04
Steel strength (structural steel)	N/mm^2	Lognormal	25	0.04–0.07

Table 2.5: Typical probabilistic load distributions (DNV 1992)

Variable	Type	Distribution
Wind	Short-term instantaneous gust speed	Normal
	Long-term n-minute average speed	Weibull
	Extreme speed, yearly	Gumbel
Wave	Short-term instant. Surface elevation (deep water)	Normal
	Short-term heights	Rayleigh
	Wave period	Longuet-Higgins
	Long-term significant wave height	Weibull
	Long-term mean zero upcrossing or peak period	Lognormal
	Extreme height, yearly	Gumbel
Current	Long-term speed	Weibull
	Extreme, yearly	Gumbel
Forces	Hydrodynamic coefficients	Lognormal

2.4.2 Need for Resistance Modelling

When conducting a reliability analysis, the two fundamental aspects of the limit-state need to be modelled: R and S. In the explanation of the method, these were limited to single variables, but in practice, each will be represented by its own equation and associated parameters. For this section, material strength is mostly considered, but the variable R can be extended to other applications including, but not limited to, flow capacity of a pipe or river, traffic capacity on a road network, soil cohesion, etc. Additionally, the variables relating to the geometry of the structure are typically classed as R variables.

To accurately assess the strength of a material, the inherent variation in material properties needs to be modelled. In general, these properties tend to have variation from point to point, and an appropriate probability distribution should be used when modelling for structural reliability. Using a probability distribution allows the specification of a mean value μ and a standard deviation σ to represent the expected value of the material property and the typical range of variation. In addition to specifying the range of values, the uncertainty surrounding the expectation of these values can also be modelled. Sources for uncertainty in material strength include:

- Deviation from sample used in testing
- Level of workmanship during construction
- How the material will respond to the environment
- The rate of deterioration for the material

These sources of uncertainty can often be mitigated against through diligent supervision and the refinement of models, and can thus be classified as epistemic, as mentioned in previous sections. The geometry of the structure is often subject to less uncertainty than the material properties, as the built structure can be measured against what is designed for. In the design phase, uncertainty can be lowered by specifying small tolerance levels, and can be further reduced in the construction phase by competent workmanship and supervision. Due to minimal expected deviation from the mean value, the basic variables relating to geometry can often be modelled as fixed or deterministic.

2.4.3 Measurement of Resistance Variables

In order to gather information about the safety of a structure, the material properties of the resistance material need to be tested against the values used in the design model. This quality control process can be achieved using two distinct approaches: total testing of the material and sample testing of the material. An example of quality control is the testing of concrete specimens during the construction phase, i.e. a concrete cube/cylinder test.

For total testing, every unit produced and is assessed on a pass/fail approach using a non-destructive testing method. This type of testing is analogous to an assembly line product inspection. For sample testing, a series of random samples are taken from the total population of produced units in order to establish a measure of quality, where each unit has an equal chance of being selected for inspection. With regard to structures, sampling efficiency can be improved by limiting the population to areas of interest; such as weak points, critical connections, etc.

2.4.4 Typical Loading Scenarios

The loads which act on structures are typically categorized as dead (permanent) and live (variable). Dead loads are those that are inherent to the geometry and composition of the structure, where live loading considers loads which are imposed upon the structure. Live loads can occur through human intervention (floor loading, traffic loading, etc.) or due to natural phenomena (wind loading, wave loading, etc.).

2.5 Probabilistic Assessment of Limit State Violation

In the undertaking a probabilistic assessment of limit-state violation, the methods used and the modelling of variables can provide inaccuracies when it comes to achieving comparable results. For the successful implementation of the method, the engineer needs to be aware of the limitations of some methods and the pitfalls of others.

2.5.1 Reliability Index and Probability of Failure

For the simple case of the FOSM method, the reliability index can be seen to be equal to the number of standard deviations the mean value lies from the failure surface. It can also be seen to be the least distance from the origin to the limit-state surface in the standardised space. Thus, as the distance from the mean to the failure surface increases, so too does the reliability index, as the failure probability decreases.

The Cornell reliability index β_C is defined as a quotient of an expected value E[G] and an uncertainty parameter D[G]. For structural reliability applications, E[G] can be modelled as the mean value μ of the parameter and D[G] can be modelled as the standard deviation σ of the parameter. So for the simple R and S scenario, the Cornell reliability index can be defined as:

$$\beta_C = \frac{\mathrm{E}[G]}{\mathrm{D}[G]} = \frac{\mathrm{E}[R] - \mathrm{E}[S]}{\sqrt{\mathrm{Var}[R] - \mathrm{Var}[S]}} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{2.12}$$

This definition of β_C accounts only for uncorrelated basic variables. For basic variables that possess a degree of correlation, as is often seen, it is necessary to modify β_C as follows (Lemaire 2009):

$$\beta_C = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2 - 2\text{cov}[R,S]}} \tag{2.13}$$

As the simple reliability case of *R* and *S* is a basic subtraction problem, it is not feasible to use the previous method to establish reliability indices for variables that are restricted to positive values. For such cases, the FOSM reliability index β_{RE} was developed (Rosenblueth and Esteva 1972) by using the logarithms of μ_Z and σ_Z in the evaluations of β.

$$\beta_{RE} = \frac{\mathrm{E}[\log(R/S)]}{\mathrm{D}[\log(R/S)]} \tag{2.14}$$

However, the limit-state is now a non-linear function, and so the first two moments cannot easily be determined. As such, the limit-state surface must be linearised in order to determine β. This can be achieved by a Taylor series expansion about one of the expected values, such as μ. This allows the limit-state function *G* and reliability function to be rewritten as:

$$G = \log\mu_R - \log\mu_S + \frac{R - \mu_R}{\mu_R} - \frac{S - \mu_S}{\mu_S} \tag{2.15}$$

$$\beta = \frac{\log\mu_R - \log\mu_S}{\sqrt{V_R^2 + V_S^2}} \tag{2.16}$$

Hasofer and Lind (1974) proposed to map the basic variables into an uncorrelated standard space, and evaluate the reliability index as the least distance from the origin of this space to the failure surface, defined by the limit-state function. This gave rise to what is now considered the design point.

2.5.2 The Concept of the Design Point

The point on the limit state surface that satisfies the condition of being minimum distance from the origin is called the design point, and is the point of maximum likelihood for the failure domain by having the greatest probability density. This point is generally used when conducting reliability studies using

FORM and SORM, as it is seen to be the optimum location for a linear expansion for nonlinear limit-state surfaces. Previously, the mean point was used for the expansion point in the linearisation, but using this method resulted in an invariance problem where equivalent limit-states were outputting different reliability indices.

As the design point was shown to be the optimum position for approximating the limit-state surface, a method needed to be developed to locate this point. Many methods have been investigated to efficiently find the design point (Liu and Der Kiureghian 1991): the gradient projection method, the penalty method, the augmented Lagrangian method, the sequential quadratic programming method, and the Hasofer-Lind and Rackwitz-Fiessler (HL-RF) method (Hasofer and Lind 1974, Rackwitz and Fiessler 1978). The most widely used algorithm to find the design point was the iterative HL-RF method, defined by the formula:

$$u_{i+1} = \left[\frac{G(u_i)}{\|\nabla G(u_i)\|} + \alpha^T u_i \right] \alpha \tag{2.17}$$

Where α^T is the transpose of α, and every successive iteration $i+1$ is a function of the previous iteration *i*. Starting from an initial expansion point, taken to be the mean value for convenience, the refined expansion point is computed using the above formula. This is repeated until the location of the design point and the reliability index stabilises. However, this method had demonstrated a convergence problem under certain circumstances, and modifications were proposed to rectify this computation issue. The most popular modification is by introducing a line search along a directional vector **d**. This can be seen as an expansion on the gradient projection method, with the improvement that the initial checking point does not need to be located on the limit-state surface.

A significant issue with using the design point as the estimate for the reliability index, and thus failure probability, is the problem of nonlinear failure surfaces that exhibit a high degree of convexity with which it is endowed. As can be seen in Figure 2.4, four limit-state functions are presented which are of different forms but possess the same design point and reliability index. However, it is obvious that the failure probability of g_1 is much more significant than that of g_4, which seems to have an outlying critical level as opposed to g_1 which exhibits a near constant critical level. The understanding of this pitfall is a mitigation of the negative effects it can have on the reliability evaluation. A procedure was proposed by Ditlevsen (1979) to rectify this issue by introducing a weighting factor ψ to the normal probability density function. The use of this method is said to provide a more robust selection of the reliability index than the HL index for nonlinear failure surfaces.

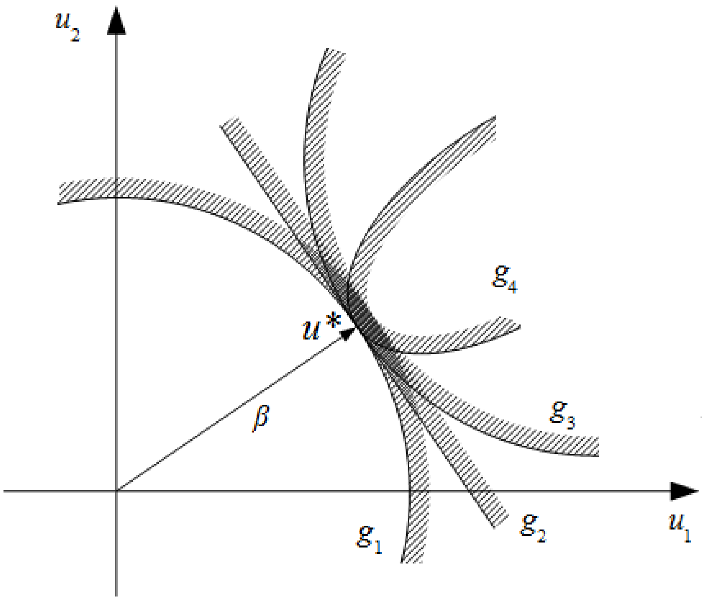

Figure 2.4: Common reliability index for differing limit-states

2.5.3 Sensitivity Studies & Parameter Importance Measures

The parametric sensitivity is the measure of change of the reliability index due to perturbations in the basic variables (Hohenbichler and Rackwitz 1986, Madsen et al. 1986). When a basic parameter θ is changed in the limit-state equation, the original failure surface $g(u, \theta) = 0$ is subject to change as a function of $d\theta$. With a new failure surface $g(u, \theta + d\theta) = 0$, the design point u^* is relocated from its original position to $u^* + du^*$ (Figure 2.5). The location of the new design point can be related to the original position through the unit directional vector α and its infinitesimal orthogonal increment $d\alpha$ (Bjerager and Krenk 1989).

$$\frac{d\beta}{d\theta} = \alpha^T \frac{du^*}{d\theta} = \frac{1}{|\nabla g(u^*)|} \frac{\partial g}{\partial \theta} \tag{2.18}$$

Sensitivity studies can be carried out within the framework of reliability analysis and it is helpful in identifying and quantifying errors in design, modelling and construction (Frangopol 1985, Nowak and Carr 1985). The importance of a variable to β is defined as the alpha-value α_i, which measures the sensitivity of β to a small variation in the mean-value μ_i of a basic random variable

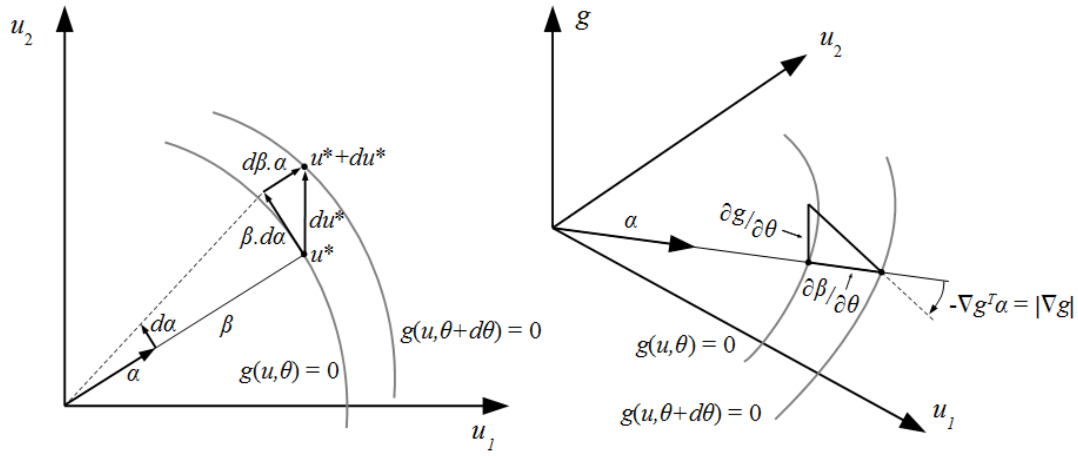

Figure 2.5: Illustration of parametric sensitivity

(Hohenbichler and Rackwitz 1986):

$$\alpha_i = \frac{\partial\beta}{\partial\mu_i} \tag{2.19}$$

This parametric sensitivity factor α_i for the reliability index β with respect to a parameter θ is defined (Madsen et al. 1986) and developed (Bjerager and Krenk 1989) as the derivative $\partial\beta/\partial\theta$. This factor measures the relative change in β due to a variation in a parameter $\Delta\theta$. For a specified or known value of $\Delta\theta$, the adjusted reliability index β' can now be expressed as:

$$\beta' = \beta + \frac{\partial\beta}{\partial\theta}\Delta\theta \tag{2.20}$$

With this expression, the magnitude of how much each parameter must change in order to satisfy a specified level of safety can be evaluated. For design, the parameters can be adjusted to obtain a target reliability index β_T, and for assessment, the parameters can be monitored such that they do not degrade to a critical reliability index β_{Min}.

$$\Delta\theta = \frac{\beta' - \beta}{\partial\beta/\partial\theta} \tag{2.21}$$

It should be noted that the terms β_T and β_{Min} are often used interchangeably, and that the rest of this chapter will use the term β_T.

As part of a sensitivity analysis, parameter importance factors α_i^2 can be determined, identifying which of the modelled parameters have the greatest impact on the reliability index, and thus, the safety of the structure.

$$\sum_{i=1}^{n} \alpha_i^2 = 1 \tag{2.22}$$

These factors indicate through their ranking, expressed as a percentage, what parameters are important for monitoring within a system and to what extent they contribute to the probability of safety or failure. Also, for varying limit states or uncertainties, the ranking of these parameters within a system can change; emphasizing the fact that the contribution of a certain factor to a failure defined by a limit state is a function of the information available about the system and the associated confidence or accuracy of that information. It should be noted that a positive α-value corresponds to a load variable and a negative α-value corresponds to a resistance variable.

The parameter importance factors allow for the computation of the omission sensitivity factor γ_i, which is the relative error of β when a stochastic variable is modelled as a deterministic parameter (Madsen 1988). This factor for a basic variable x_i is measured as the inverse ratio of β and an adjusted reliability index β' when the random variable x_i is replaced by a deterministic parameter, typically its median (Ditlevsen and Madsen 1996).

$$\gamma_i = \frac{1}{\sqrt{1 - \alpha_i^2}} \tag{2.23}$$

2.6 Time Dependent Reliability

2.6.1 Concept of Time Dependence

A powerful application of the structural reliability method is the ability to perform a time-dependent reliability analysis; whereby the reliability of a structure can be predicted at a certain point in time in the future, or over a specific time interval. This is often necessary when considering a life-cycle approach to the design or assessment of structures, as the condition of a structure is likely to vary with time. Advanced applications of the method concern the evaluation of fatigue effects or the dynamic application of loads, but more fundamental topics are concerned with changes in the basic variables which govern the limit-state equations. In a typical example of the life of a structure, it is often observed that the structural resistance decreases over time due to destructive processes such as corrosion, section loss etc.; and the applied load often increases due to increased demand or unfavourable change of use. Thus, the relevant basic variables can be considered as functions of time and that the structural reliability decreases with time, typically. It should be noted that factors which improve the lifetime reliability include structural strengthening or favourable change of use.

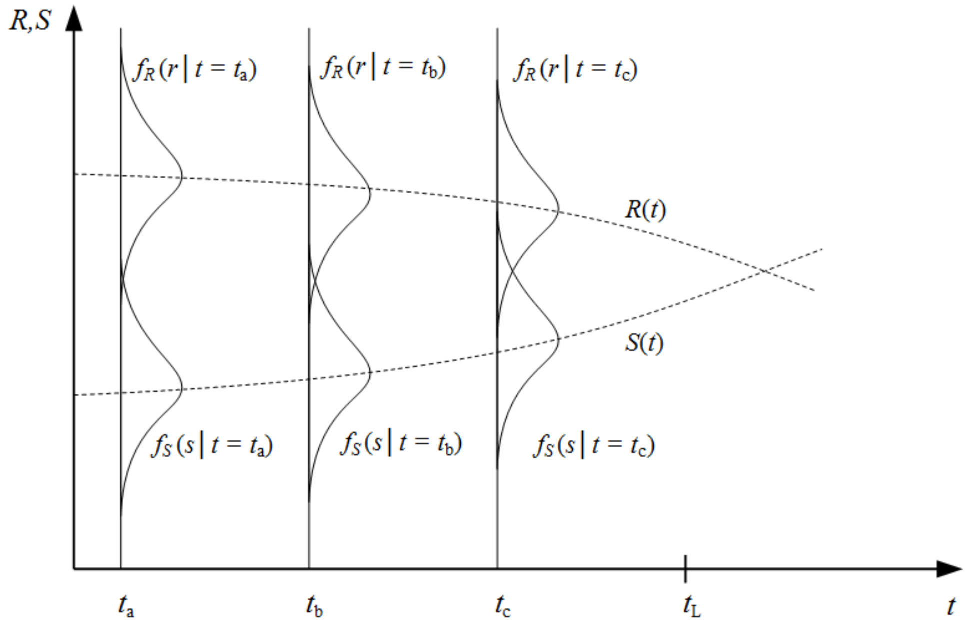

Figure 2.6: General time-dependent reliability problem

2.6.2 Handling Time Dependency in Reliability Analysis

The time-dependent reliability model extends the fundamental model previously discussed by introducing the time function t:

$$P_f(t) = P[R(t) \leq S(t)] \tag{2.24}$$

$$P_f(t) = \int_{G[\mathrm{X}(t)]\leq 0} f_{\mathrm{X}(t)}[\mathrm{x}(t)]\mathrm{d}\mathrm{x}(t) \tag{2.25}$$

Using the above equations, the failure probability can be calculated for a specific time t or can be evaluated as the probability of failure occurring over a time period ending at t. To achieve this, integrate the above over the bounds 0 (now) to t. This time period can be defined to be the life of the structure, with the bound t_0 to t_L. It is also possible to evaluate the first instance where the load exceeds the resistance, or where $S(t)$ "crosses" $R(t)$. The evaluation of this point, known as the upcrossing point or barrier crossing point, is done using stochastic process theory, which is discussed in other textbooks in greater detail (Melchers 1999).

From a time-dependent reliability assessment, a reliability index profile can be created over a period of time (Kong and Frangopol 2003). This profile plots the change in reliability index β over time due to degradation and completed intervention actions. The new profile is obtained by superimposing the profile of expected degradation or interventions onto the existing profile obtained from a time-dependent reliability assessment (Figure 2.7).

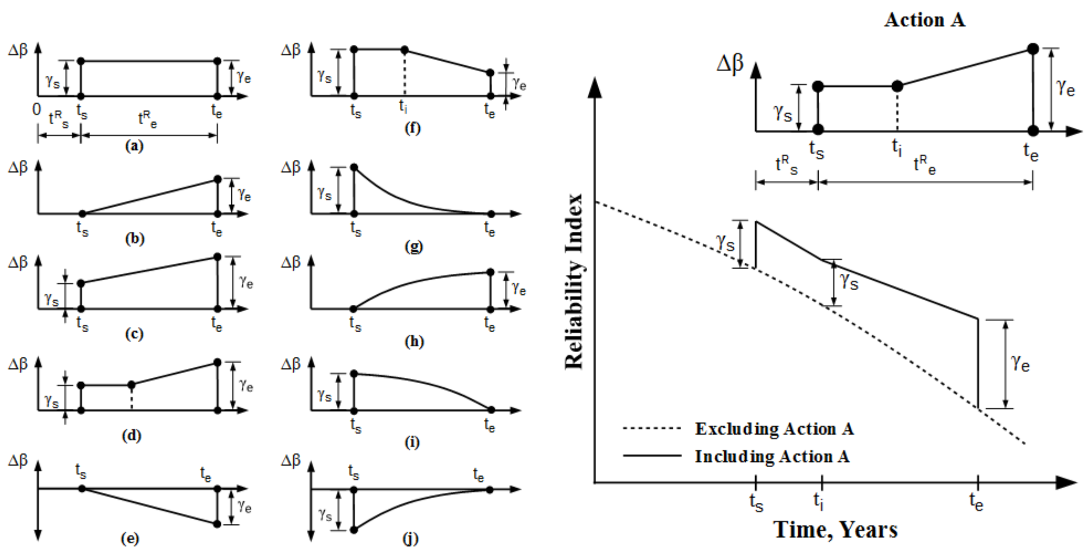

Figure 2.7: Various action-based reliability index profiles and their effect on existing reliability profiles, adapted from Kong and Frangopol (2003)

$$\beta_j(t) = \beta_{j,o}(t) + \sum_{i=1}^{n} \Delta\beta_{j,i}(t) \tag{2.26}$$

Where $\Delta\beta_{j,i}(t)$ is the additional reliability index profile generated by the action i, and n is the number of actions associated with the failure mode j during the lifetime. Actions associated with a value for $\Delta\beta$ greater than unity are those that positively contribute to the structural safety; examples of which include maintenance activities or physical effects such as concrete hardening. Conversely, values for $\Delta\beta$ less than unity are those that negatively contribute to structural safety; examples of which are degradation over time or sudden effects such as damage from an impact load. This is useful for performance-based design as it enables the evaluation of how much damage a structure can sustain at a single point in time before becoming critically unsafe.

2.6.3 Time-Dependent Deterioration Modelling

The use of time-dependent reliability assessment facilitates the prediction of future performance by including information obtained from future deterioration modelling of the structure. This process is used later in the thesis to quantify the performance of a number of sample bridges, and the methods used to model this deterioration are briefly explained here.

Here we will consider the chloride-induced deterioration of a reinforced concrete under the three main phases of deterioration: time to corrosion initiation T_i, time to crack initiation T_{1st}, and time to crack propagation T_{cp} (Kenshel and O'Connor 2009). Chloride-induced corrosion is among the most widespread deterioration mechanisms for reinforced concrete structures, and its presence is typically indicated by the cracking of the concrete cover to reinforcement (O'Brien et al. 2015a). This occurs due to the expansive nature of corrosion inducing a tensile stress in the concrete surrounding the reinforcement bars. The time at which corrosion will first occur T_i can be determined by Fick's second law of diffusion:

$$T_i = \frac{C^2}{4D_{app}}\left[\mathrm{erf}^{-1}\left(\frac{C_s - C_{cr}}{C_s}\right)\right]^{-2} \tag{2.27}$$

Where D_{app} is the apparent diffusion coefficient (mm^2/year); C_s is the surface chloride concentration (% per weight of cement or concrete); C_{cr} is the critical chloride concentration (% per weight of cement or concrete); and C is the concrete cover (mm). The time (years) from this initiation of corrosion to the first instance of cracking T_{1st} can be determined from a number of numerical models, such as (El Maaddawy and Soudki 2007):

$$T_{1st} = \left[\frac{7117.5(D+2\delta_o)(1+\nu+\psi)}{i_{corr}E_{ef}}\right]\left[\frac{2Cf_{ct}}{D}\frac{2\delta_o E_{ef}}{(1+\nu+\psi)(D+2\delta_o)}\right] \tag{2.28}$$

Where D is the diameter of the steel rebar (mm); δ_o is the thickness of the porous zone around the steel bar which will have to be filled before the tensile stresses can be generated (mm); ψ is a factor dependent on D, C and δ_o; i_{corr} is the corrosion rate density (μA/cm^2); E_c is the elastic modulus of concrete; E_{ef} is the effective elastic modulus of concrete that is equal to $[E_c/(1+\phi_{cr})]$, where

ϕ_{cr} is the concrete creep coefficient; ν is the Poisson ratio for concrete; and f_{ct} is the tensile strength of the concrete. In the final phase of chloride-induced corrosion, the time (years) from the first instance of cracking to the maximum allowable cracking is (Vu and Stewart 2005):

$$T_{cp} = 0.0167 i_{corr}^{-1.1} \left[42.9 \left(\frac{\mathrm{wc}}{C} \right)^{-0.54} + \left(\frac{w_{lim} - 0.3}{0.0062} \right)^{1.5} \right] \tag{2.29}$$

Where wc is the water/cement ration; and w_{lim} is the maximum crack size. The ability to be able to determine the time in which the structure is expected to develop critical cracking enables a more accurate estimation of the future reliability of the structure. In addition to modelling this critical crack propagation, it is also possible to account for the expected loss of an effective area in a time-dependent reliability model. Two models for reinforcement section loss are often considered: uniform corrosion and pitting corrosion, for which numerous methodologies exist for the calculation of section loss (Andrade et al. 1993, Val and Melchers 1997). As section loss increases, the structural capacity of critical structural elements is compromised and thus the reliability of the structure is also compromised. This section loss can be modelled using the stochastic methods mentioned previously. The application of these methods will be seen in Chapter 4, whereby single point in time estimations of section loss be will evaluated and used to compute reliability indices at these times. While this represents only one method of computed time dependent reliability, it is adequate to reach the objectives of this thesis.

2.7 Conclusion

In this chapter, a performance-based approach to the design and assessment of structures was presented through the implementation of the probabilistic reliability method. The basis of the method was explained and shown to be present in modern structural design codes of practice. The rationale behind the use of this method is how it allows the stochastic modelling of variables in the limit-state design, which is more reflective of actual structural realization than the standard deterministic approach. The various reliability methods used to compute the safety classification of structures were shown, and guidance was given on which method should be chosen in response to the requirement of the structure.

The modelling of the basic variables for load and resistance was presented, and the sensitivity of the method to input parameters was highlighted, along with the potential advantageous by-products of using the method; such as parametric sensitivity and parameter importance measures. The practical hindrances to the widespread adoption of the method have been resolved in the development of software applications and the continued efforts to improve the robustness of the method. In Chapters 3 and 4, the effect that disparate information levels for load and resistance modelling have on β will be shown.

Chapter 3

Reliability Analysis with Uncertain Parameters

3.1 Introduction

3.1.1 Overview

In the previous chapter, a background has been presented to the structural reliability method, from which it can be seen the role that small changes on the basic input model can have on the computed reliability of the structure. With regard to our basic model formulation of Equation 1.1, this chapter will investigate the uncertainty in information surrounding the resource variable R. By increasing the uncertainty in the basic variables, the level at which reliability analysis is compromised to a point that it becomes difficult to extract any valuable information from the analysis can be estimated. From the analysis, it can be seen that it is possible to identify the most important parameters that influence the reliability index of a particular bridge type; however, these calibrations become clouded when more uncertainty in information is introduced to the model.

Presented here is a reliability analysis of three bridges; comprising reinforced concrete slab, reinforced concrete beam-slab, and prestressed concrete beam construction, with a focus on the sensitivity analysis and the analysis of parameter importance measures. The basis of the analysis stems from the possibility of investigating similarities in various parameters, leading to the establishment of network-level indicators based on fully probabilistic assessments; as these bridges are typical of construction details on road networks. A probabilistic analysis is conducted, taking uncertainty in relation to the available information into account. Parametric importance measures are established across the three bridge types, and patterns identified from these studies suggest the potential for reliability-based network calibrations of bridge structures.

3.1.2 Background

As bridge infrastructure networks age, it is often necessary to employ non-deterministic techniques in the assessment of intervention options for deteriorating network assets to maintain an adequate level of safety throughout the network (Žnidarič et al. 2011). Probability concepts have been shown to have significant advantages in the design and assessment of engineering structures, specifically structural reliability methods (Ang and Tang 2007). A reliability-based approach for quantifying the safety of structures enables a lifetime evaluation of both individual and networks of structures (Akgül and Frangopol 2004a,b, Frangopol and Das 1999, Liu and Frangopol 2006a,b, Frangopol and Liu 2007a,b, Frangopol 2011, Bocchini and Frangopol 2011a,b,c, Saydam et al. 2013). While this method is commonly implemented at both a component and system level for an individual bridge in isolation (O'Connor and Enevoldsen 2008, Estes and Frangopol 2001a), there are advantages to conducting a reliability analysis for a network of bridges (Frangopol and Bocchini 2012); highlighting critical components and providing the stakeholders of bridge stock with comparable safety indices and sensitivity measures (O'Connor and Enevoldsen 2007, Dong et al. 2014).

The effective allocation of capital resources seeks to minimise the inherent risks associated with investments through the use of advanced methods (Mueller and Stewart 2011). Reliability methods are an effective tool for the monitoring of the asset base and, thus, allowing the prioritisation of intervention and investment requirements in a more careful and rational manner. Intervention can be focused to address the most important parameters that govern the safety of the bridges, as highlighted by the parametric sensitivity and parameter importance factors, which are beneficial by-products of reliability assessments. Conducting this analysis over a network allows for the comparison of different parameters and uncertainties in each bridge type, and investigates correlations that arise between them (Hanley and Pakrashi 2014). This emphasizes the need for a network based calibration of the importance of certain critical parameters, and provides a framework for future assessments of the structures.

The objective of this chapter is to investigate how uncertainty in the parameters involved can affect this proposed framework and to assess the existence of a minimum level of confidence that is required in order to make a rational intervention decision. A number of bridges are described and results of the reliability analysis are shown over the group and the parametric sensitivity studies are detailed; highlighting critical parameters that contribute to the violation of the established limit-states. Investigations are carried out to obtain common markers or patterns of information present in the bridges described by the sensitivity studies and parameter importance measures. These measures provide greater information for an engineer in terms of how to assemble a probabilistic model, and guides the process of further data gathering through other assessment methods, in an effort to build a more representative model from which to base intervention decisions.

3.2 Description of Bridges

In order to assess the effects of uncertainty on the reliability analysis of a number bridges and bridge networks, three single span, simply supported bridges were evaluated as a case study. The bridges used for the analysis have reinforced concrete slab, reinforced concrete beam-slab, and prestressed concrete beam construction; of which is largely seen in national bridge stock in Ireland (Duffy 2004) and mainland Europe (Žnidarič et al. 2011). The general arrangement and cross-sections of these bridges can be seen in Figure 3.1, and they all comprise a span length of 16m (Table 3.1). This span was chosen with regard to the available probabilistic load model, as detailed in the following section. The cross-section parameters and structural information can be seen in Tables 3.2 & 3.3.

3.3 Assessment Methodology

In this assessment, the flexural limit-state g was analysed; having been identified as the critical limit-state in recent assessments carried out in Ireland (NRA 2010). The flexural capacity M_u was tested against the bending moment effects of the self-weight of the bridge M_{DL}, the superimposed dead load of the road

Table 3.1: General bridge dimensions

Attribute	Value
Number of spans (No.)	1
Overall length (m)	16
Width out-to-out (m)	10.4
Width of footway (m)	1.5
Width of carriageway (m)	6.4
Road surface (mm)	100
Number of lanes (No.)	2

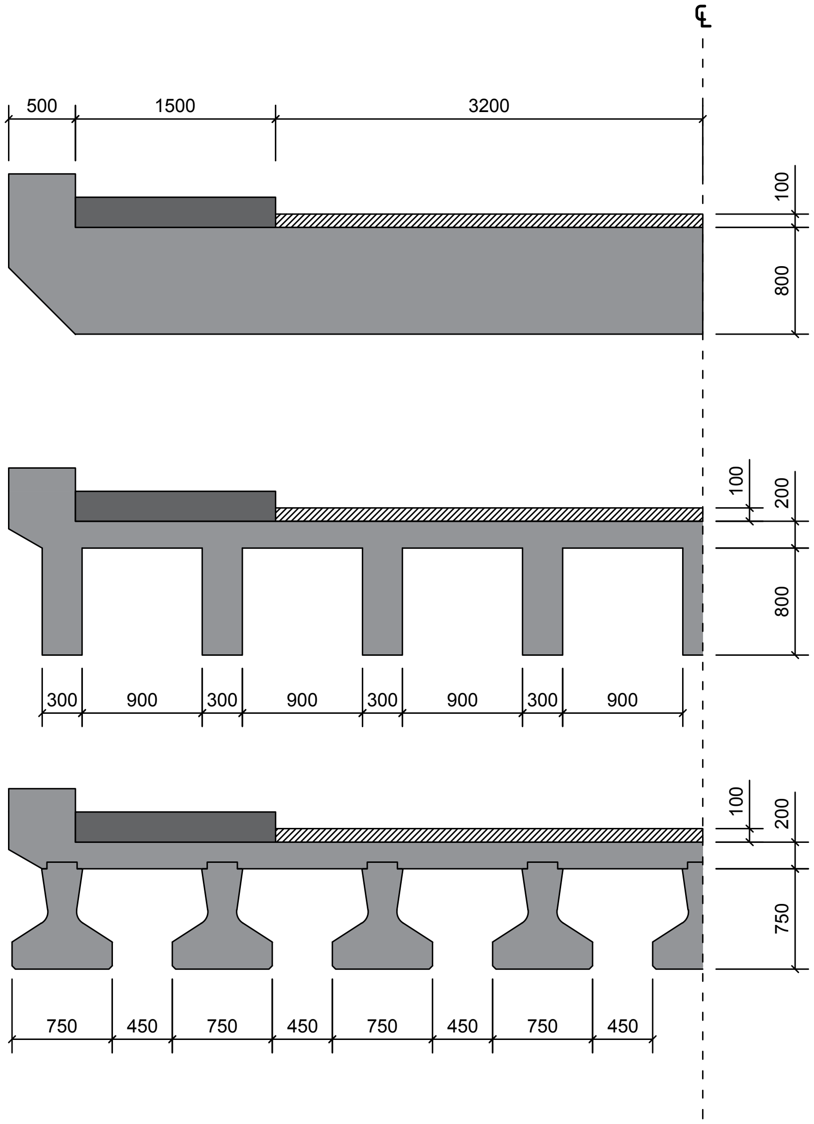

Figure 3.1: General arrangement of bridges under analysis.

surface M_{SDL}, and the various bending moments produced by changing traffic load specifications M_{LL}.

$$g = R - S = M_u - M_{DL} - M_{SDL} - M_{LL} \tag{3.1}$$

The probabilistic load model used in this chapter was developed by Chryssanthopoulos et al. (1997) and Cooper (1997), and was derived as a static load model with a uniformly distributed load (UDL) of 27 kN/m and 2 axle loads of 300 kN each, factored by a statistically defined variable λ_{Prob} with a Gumbel distribution; extrapolated from WIM data on motorway bridges in the UK. The variable λ_{Prob} given in Table 3.2 corresponds to a major road that experiences a traffic volume flow per direction per day of 10,000 (Cooper 1997); with the statistical parameters defined in Table 3.2 being one year parameters for a Gumbel distribution.

For computational efficiency, the limit state equations are expressed in parametric form (Akgül and Frangopol 2004b), whereby the random variables X_{ij} and the deterministic parameters Y_{ij} are decoupled, and groups of Y_i are combined into deterministic constant coefficients C_{ij} in the limit state equations. For the reinforced concrete slab under consideration, the limit state equation for flexural failure is defined as:

$$g_{slab,m} = \left(C_{01} A_s f_y \gamma_m \lambda_d - C_{02} \frac{A_s^2 f_y^2 \gamma_m}{f_c} \right) - C_{03}\lambda_c - C_{04}\lambda_s - C_{05}\lambda_{Prob} \tag{3.2}$$

where the random variables A_s, f_c, f_y, and the uncertainty factors λ_x and γ_m are defined in Table 3.2, and the deterministic constant coefficients C_{ij} are functions of the deterministic parameters defined in Table 3.3, where:

$$C_{01} = \frac{d}{1000000}$$

$$C_{02} = \frac{1}{1200000b}$$

$$C_{03} = \frac{\rho_c b h_c L^2}{8000000}$$

$$C_{04} = \frac{\rho_s b t_s L^2}{8000000}$$

$$C_{05} = \left\{ \frac{27L^2}{8} + \left(\frac{300\left[\left(\frac{L}{2} + 0.3\right) + \left(\frac{L}{2} - 0.9\right)\right]}{L} \right) \left(\frac{L}{2} - 0.3 \right) \right\} \frac{b}{1000b_L}$$

For the reinforced concrete beam bridge, the flexural limit state is defined similarly as:

$$g_{beam,m} = \left(C_{11} A_s f_y \gamma_m \lambda_d - C_{12} \frac{A_s^2 f_y^2 \gamma_m}{f_c} \right) - C_{13}\lambda_c - C_{14}\lambda_s - C_{15}\lambda_{Prob} \qquad (3.3)$$

where, in this case, the deterministic constant coefficients C_{ij} are defined as:

$$C_{11} = C_{01}$$

$$C_{12} = C_{02}$$

$$C_{13} = \frac{\rho_c \left(b_{eff} h_f + [h_c - h_f] \, b_w \right) L^2}{8000000}$$

$$C_{14} = \frac{\rho_s b_{eff} t_s L^2}{8000000}$$

$$C_{15} = \left\{ \frac{27L^2}{8} + \left(\frac{300 \left[\left(\frac{L}{2} + 0.3 \right) + \left(\frac{L}{2} - 0.9 \right) \right]}{L} \right) \left(\frac{L}{2} - 0.3 \right) \right\} \frac{b_{eff}}{1000 b_L}$$

Lastly, the flexural limit state for the prestressed concrete beam bridge is defined as:

$$g_{pres,m} = \left(C_{21} A_{ps} C_{23} f_{pu} \gamma_m \lambda_d - C_{22} \frac{A_{ps}^2 C_{23} f_{pu}^2 \gamma_m}{f_c} \right) - C_{24}\lambda_c - C_{25}\lambda_s - C_{26}\lambda_{Prob} \qquad (3.4)$$

and the deterministic constant coefficients C_{ij} are defined as:

$$C_{21} = C_{11} = C_{01}$$

$$C_{22} = C_{12} = C_{02}$$

$$C_{23} = 0.7 \times 0.7 = 0.49$$

$$C_{24} = \frac{\rho_c \left(A_b + b_{eff} h_f - 0.2884 t_o \left[h_c - h_f + t_o \right] - 3.75 \right) L^2}{8000000}$$

$$C_{25} = C_{14}$$

$$C_{26} = C_{15}$$

where C_{23} is a factor account for the effects of relaxation and creep.

Table 3.2: Random variables for all bridges (All RV's have lognormal distributions (Akgül and Frangopol 2005b, 2004c), with the exception of λ_{Prob}, which has a Gumbel distribution (Cooper 1997))

Tag	Variable	Description	μ	σ
X_{01}	A_s	Area of flexural steel reinforcement (mm^2)	6835.35	341.7675
X_{02}	f_{cu}	Compressive strength of concrete (N/mm^2)	50	7.5
X_{03}	f_y	Yield strength of reinforcing steel (N/mm^2)	500	50
X_{04}	γ_m	Model uncertainty for flexure	1	0.1
X_{05}	λ_c	Concrete weight uncertainty factor	1	0.1
X_{06}	λ_s	Surfacing weight uncertainty factor	1	0.25
X_{07}	λ_d	Effective depth uncertainty factor	1	0.02
X_{08}	λ_{Prob}	Probabilistic load adjustment factor	0.4101	0.02466
X_{11}	A_s	Area of flexural steel reinforcement (mm^2)	5192.69	259.6345
X_{12}	f_{cu}	Compressive strength of concrete (N/mm^2)	50	7.5
X_{13}	f_y	Yield strength of reinforcing steel (N/mm^2)	500	50
X_{14}	γ_m	Model uncertainty for flexure	1	0.1
X_{15}	λ_c	Concrete weight uncertainty factor	1	0.1
X_{16}	λ_s	Surfacing weight uncertainty factor	1	0.25
X_{17}	λ_d	Effective depth uncertainty factor	1	0.02
X_{18}	λ_{Prob}	Probabilistic load adjustment factor	0.4101	0.02466
X_{21}	A_p	Area of prestressing steel (mm^2)	3892	194.6
X_{22}	f_{cu}	Compressive strength of concrete (N/mm^2)	50	7.5
X_{23}	f_{pu}	Prestressing steel strength (N/mm^2)	1670	83.5
X_{24}	γ_m	Model uncertainty for flexure	1	0.1
X_{25}	λ_c	Concrete weight uncertainty factor	1	0.1
X_{26}	λ_s	Surfacing weight uncertainty factor	1	0.25
X_{27}	λ_d	Effective depth uncertainty factor	1	0.02
X_{28}	λ_{Prob}	Probabilistic load adjustment factor	0.4101	0.02466

Note: Slab = X_{0i}; Beam = X_{1i}; Prestressed = X_{2i}

3.4 Results

3.4.1 Reliability Indices

The reliability analysis used herein was conducted using OpenSees, an open-source software framework for evaluating the performance of structural systems (McKenna et al. 2002). The reliability indices β for the bridges within the network were determined using FORM and were then checked for non-linearity using SORM. The high correlation between β_{FORM} and β_{SORM} suggested that the failure surfaces were highly linear. The results for the three bridges were

Table 3.3: Deterministic parameters for all bridges

Bridge	Tag	Parameter	Description	Value
Slab	Y_{01}	b	Width of section considered (mm)	1000
	Y_{02}	b_L	Notional lane width (m)	3.2
	Y_{03}	d	Effective depth of section (mm)	724
	Y_{04}	L	Span length (m)	16
	Y_{05}	h_c	Height of concrete slab (mm)	800
	Y_{06}	t_s	Thickness of road surface (mm)	100
	Y_{07}	ρ_c	Self-weight on concrete (kN/m^3)	25
	Y_{08}	ρ_s	Self-weight of surface (kN/m^3)	24
Beam	Y_{11}	b_{eff}	Effective flange width (mm)	1200
	Y_{12}	b_L	Notional lane width (m)	3.2
	Y_{13}	b_w	Width of beam (mm)	300
	Y_{14}	d	Effective depth of section (mm)	924
	Y_{15}	L	Span length (m)	16
	Y_{16}	h_c	Overall height of concrete beam (mm)	1000
	Y_{17}	h_f	Thickness of concrete flange/slab (mm)	200
	Y_{18}	t_s	Thickness of road surface (mm)	100
	Y_{19}	ρ_c	Self-weight on concrete (kN/m^3)	25
	Y_{110}	ρ_s	Self-weight of surface (kN/m^3)	24
Prestressed	Y_{21}	A_b	Area of precast section (mm^2)	339882
	Y_{22}	b_{eff}	Effective flange width (mm)	1200
	Y_{23}	b_L	Notional lane width (m)	3.2
	Y_{24}	d	Effective depth of section (mm)	818.571
	Y_{25}	L	Span length (m)	16
	Y_{26}	h_c	Overall height of section (mm)	950
	Y_{27}	h_f	Thickness of concrete flange/slab (mm)	200
	Y_{28}	t_o	Thickness of overlap (mm)	50
	Y_{29}	t_s	Thickness of road surface (mm)	100
	Y_{210}	ρ_c	Self-weight on concrete (kN/m^3)	25
	Y_{211}	ρ_s	Self-weight of surface (kN/m^3)	24

grouped together to determine if a relationship existed for β between common bridge materials, and to what degree uncertainty, with regard to the random variables, affected the results of a safety classification based on β. The uncertainty in the random variables is represented by the coefficient of variation (CoV), which is a relative measure of dispersion within the probability density function (PDF). It is proposed that a high level of certainty for the value of a random variable would, as much as practicable, manifest itself as a PDF with a narrow dispersion. For bridge structures, this can be seen to occur in material strengths where a variation is present, and the level of variation across the structure can be known or unknown; based on the level of information obtained from methods such as non-destructive testing (NDT) or structural health monitoring (SHM) (Frangopol 2011, Frangopol and Bocchini 2012). As they can be measured with a high degree of accuracy, the basic variables related to the bridge geometry are considered deterministic (Table 3.3), and those related to material properties were modelled stochastically (Table 3.2).

The results obtained from the analysis based on the values given Tables 3.2 & 3.3 yielded reliability indices of $\beta_{slab} = 3.68$, $\beta_{beam} = 4.79$, and $\beta_{pres} = 4.93$. Now, these reliability indices were obtained based on the mean values μ of the random variables X_{ij} and their variable standard deviations σ; indicating various levels of uncertainty surrounding these random variables. However, to investigate the effect that uncertainty in model information has on the results of a reliability analysis, the assessment was conducted for increasing uniform uncertainty in the random variables. In this study, the same CoV has been chosen for all random variable with the exception of λ_{Prob}, for which the probabilistic parameters remained unchanged. This can be seen in Figure 3.2, where it is observed that β for all bridges decreases at a divergent rate for uniform CoVs of 0.05 to 0.4. It can be seen that for a uniform CoV of 0.05, β for the slab, beam-slab, and prestressed concrete bridge is 6.10, 7.53, and 7.13, respectively. For a uniform CoV of 0.1, these values decrease to 3.05, 4.00, and 3.69, respectively. It can be seen that these values are less than that for the analysis under variable values of CoV for different σ, but it is notable that the prestressed bridge no longer has the highest β. This can be attributed to the greater affect that a CoV of 0.1 has on this model, as the initial analysis had CoV values in the order of 0.05, due to the nature of the manufacture of prestressed concrete elements and greater control over material quality. Conversely, the reinforced concrete beams were initially analysed with a CoV of 0.1 being typical the random variables, based on the in-situ nature of the construction. As the CoV increased to 0.4, it can be seen that the values of β decreased to 0.59, 0.80, and 0.73 for the slab, beam, and prestressed bridge, respectively.

It is expected and observed that β decreases with increasing CoV. It can be seen that this decrease occurs almost as a divergent series from a CoV of 0.05 to 0.1, 0.2, 0.3, and 0.4 at an approximate rate of 1, ${}^{1}/_{2}$, ${}^{1}/_{3}$, and ${}^{1}/_{4}$, respectively. This demonstrates the importance of limiting uncertainty within the probabilistic model, as the greatest decrease occurs in the CoV increase from 0.05 to 0.1, after which it is observed that the decrease in β slows. Consequently, for higher levels of uncertainty, the lack of information governs the estimated risk of failure and this also explains the lack of variability of different bridges at higher

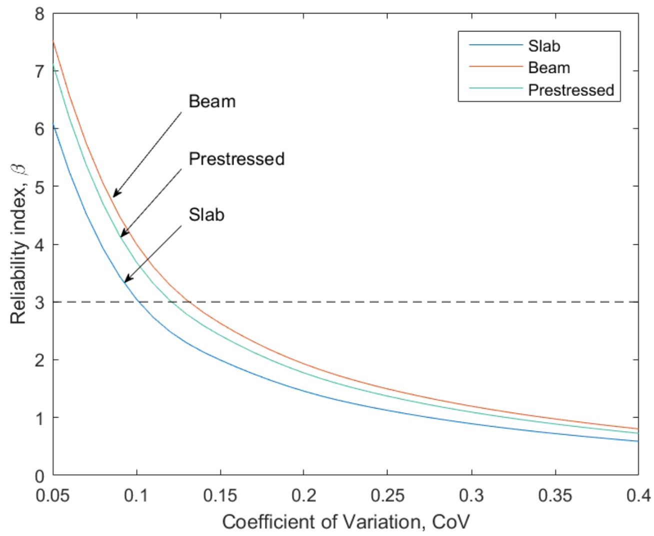

Figure 3.2: Reliability index for increasing levels of CoV.

levels of uncertainty. This builds the basis for assessing the relative importance of the basic variables in the failure surface, as it shows that there can be significant discrepancies in β based on small changes in CoV, especially for initial estimates.

3.4.2 Sensitivity Studies

Sensitivity assessments were carried out by considering a 10% perturbation in the various parameters involved in the assessment, which demonstrate the relative contribution each basic variable makes to β (DNV 1992). The resulting change in reliability index $\Delta\beta$ can be seen in Figure 3.3 for each of the three bridges under assessment. From this, it can be seen that the variable X_{i4}, corresponding to the model uncertainty for flexure γ_m, has the greatest positive contribution to β for all bridges. This positive contribution is expected considering it forms part of the R component of our general limit state equation. The next highest positive contributors correspond to $X_{i7}(\lambda_d)$, $X_{i3}(f_{y,pu})$, and $X_{i1}(A_{s,p})$. As X_{i7} is used to represent the deterministic parameter of effective section depth d, a mean increase in X_{i7} is akin to a general increase in section depth; and while providing greater stiffness to a particular section, it represents more of a consideration in the design of these cross-sections than in the assessment of such. For X_{i3} and For X_{i1}, it can be seen that these variables contribute significantly to β and so significant effort should be used in reducing the level of uncertainty associated with these variables when developing an assessment model. It is noteworthy that the compressive strength of concrete $f_{cu}(X_{i2})$ offers little to computation of β for this limit-state, and thus resources need not be deployed into reducing the uncertainty in this variable in model building.

It can be seen that the three load variables X_{i5}, X_{i6}, and X_{i8} are those which negatively contribute to β, as expected, but that the predominantly negative variable is X_{i5}, corresponding to the weight of concrete. Here, it can also be seen that this represents the greatest deviation in $\Delta\beta$ across the three bridges; which have largely corresponded in value for the other variables. Here, the slab bridge is the one that is most adversely affected by this variable, and this is due to the higher volumes of concrete used in this type of construction relative to the others. It can also be seen that the prestressed concrete bridge is also slightly more affected by this variable, as this type of construction here has beams of greater area than the beam bridge.

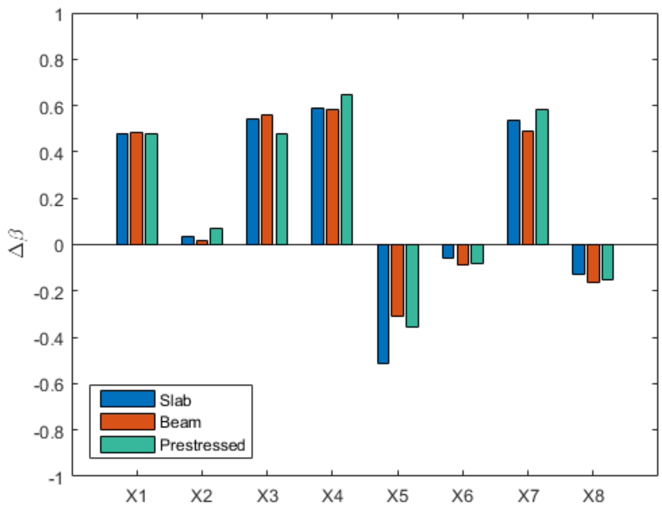

Figure 3.3: Change in reliability index considering a 10% increase in basic variables.

Now, while this physical reasoning has provided a good verification on the veracity of the presented model, it is important to investigate how sensitive this model is to increasing levels of uncertainty across the basic variables; as before. For the slab bridge alone, the resulting change in reliability index $\Delta\beta$, considering a 10% perturbation and for increasing levels of CoV, can be seen in Figure 3.4. For a uniform change in CoV, it can be seen that the general relationship between the variables remains as before, and that the increase in CoV merely lowers $\Delta\beta$ in magnitude, corresponding to the lower values of β seen in Figure 3.2 for higher values of CoV.

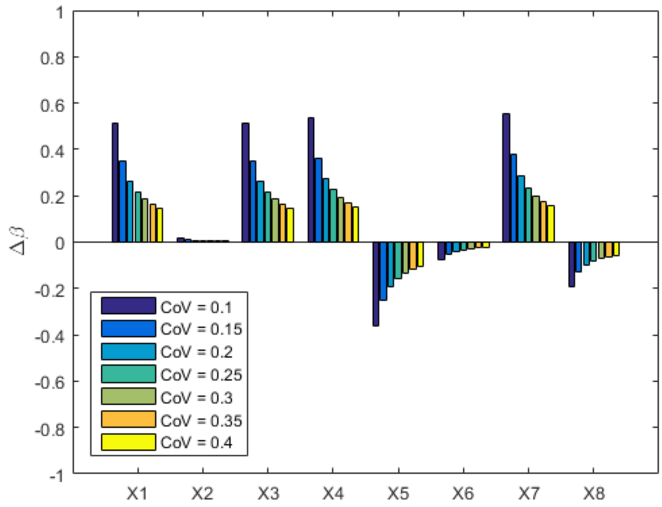

Figure 3.4: Change in reliability index for reinforced concrete slab considering a 10% increase in basic variables for different levels of CoV.

Again, it can be seen that $\Delta\beta$ decreases in the same divergent series manner as β from before. This shows that while it is important to accurately model the uncertainty in the random variables, as much as possible, that the overall relationship between the basic variables is invariant to this. Such a relationship may only change if there was significant variation in uncertainty between important basic variables, but modelling a reliability analysis under these conditions would fall out of line with best practice.

Finally, the relationship between sensitivity of the models under variable uncertainty and uniform uncertainty can be seen in Figure 3.5 for all bridges; where the uniform uncertainty is only presented for a CoV of 0.1, due to the observed relationship for subsequently increased values of CoV. Here it can be seen that there is general agreement between the values of $\Delta\beta$ between the variable and uniform uncertainty. However, this agreement in magnitude is largely a function of the values of β for these assessments, from which a uniform CoV of 0.1 was similar to that of the initial assessment.

3.4.3 Importance Factors

Importance factors α_i^2 were determined to allow the relative ranking of random variables to aid the assessment process. These factors highlight those random variables which have the greatest influence on β, and thus which variables it

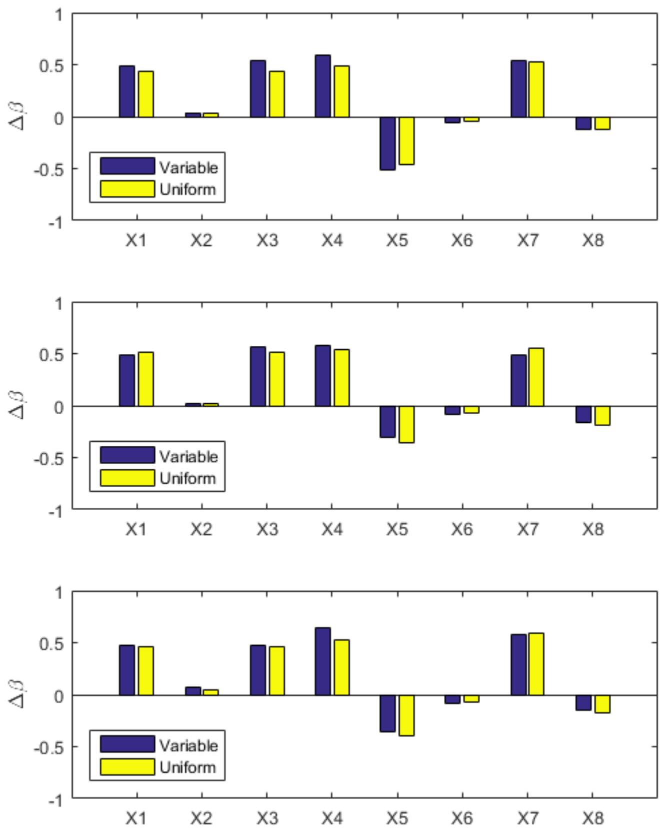

Figure 3.5: Change in reliability index for considering a 10% increase in basic variables under variable and uniform uncertainty for (a.) slab, (b.) beam, and (c.) prestressed bridges.

would be beneficial to reduce the level of uncertainty. Random variables with low importance factors can afford to be modelled as deterministic parameters, without significant change in the computed β. Those with high importance factors should be prioritised when more detailed material assessments are deemed necessary.

For the reinforced concrete slab bridge, it can be seen that the variable with the highest important factor is $X_{i5}(\lambda_c)$ for the initial assessment (Figure 3.6). This correlates with the result seen for the parametric sensitivity in the previous sections, and is due to the fact that the slab bridge contains a larger portion of concrete per unit width than a beam bridge, and thus the unfavourable effect of the self-weight of the bridge is more pronounced; as can be seen with the corresponding important factors for X_{i5} in the reinforced concrete and prestressed beam bridges. Conversely, it can be seen that the other component of the bridge self-weight, in the form of the superimposed permanent load of the road surface X_{i6} is seen to be more of a significant factor for these two bridges, and it is deemed largely unimportant for the slab bridge. Further remaining large importance factors are seen for the variables $X_{i3}(f_{y,pu})$ and $X_{i3}(\gamma_m)$, however less importance is seen on the variable $X_{33}(f_{pu})$ for the prestressed concrete bridge. For all bridges, $X_{i2}(f_{cu})$ has a very low importance factor and can modelled deterministically without much adverse effect on the reliability model. When allocating resources for determining information to be included in the assessment model, these results would suggest that any chemical inspections or non-destructive testing of these bridge types should focus entirely on evaluating an accurate model for the PDF of $A_{s,ps}$ and f_{pu}, and it would be considered unnecessary to establish anything more than initial estimates of the properties of f_{cu}. This can drive how destructive and non-destructive evaluation of bridges are conducted by prioritising tests based on the level of detail they produce regarding these variables.

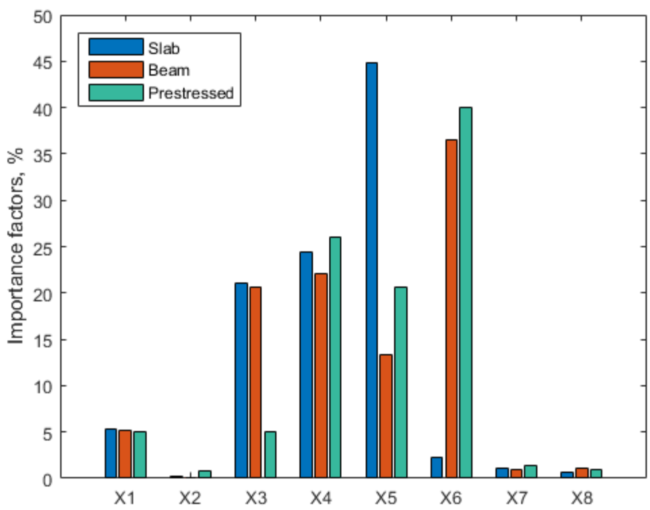

Figure 3.6: Importance factors for probabilistic variables.

When evaluating the importance factors for a uniform increase in the level of CoV in the random variables, the same relative change was seen in the results as observed for β and $\Delta\beta$ previously. When comparing the importance factors for the initial assessment with variable values of CoV to an assessment with a uniform CoV of 0.1, it can be seen that while there is broad agreement in the importance factors in some variables between both assessments, there is significant discrepancy in a number of variables (Figure 3.7). This is primarily seen in the variables $X_{i6}(\lambda_s)$ and $X_{i7}(\lambda_d)$, which display significant differences between the initial assessment and the uniform CoV of 0.1 assessment. When considering the probabilistic parameters used in the initial assessment (Table 3.2), it can be seen that X_{i6} has an initial CoV of 0.25 and that X_{i7} has an initial CoV of 0.02. As X_{i6} was modelled with more uncertainty in the initial assessment, it is acknowledged that the stochastic importance of this variable is reduced when it is modelled with a lower uncertainty attached. Similarly, now that X_{i7} has had a relatively significant increase in CoV, this variable now demonstrates more stochastic importance than before. In fact, it can be argued that X_{i6} was initially modelled with too large of a CoV of 0.25, with the next highest CoV being 0.1; and thus the large importance factor for this variable guides us to re-evaluate how this variable should be modelled in the reliability assessment. This also demonstrates some qualitative significance in that the surface weight of the road is not expected to contribute this significantly to the structural model. Thus, as the quality of information included in the analysis has significant bearing on the variable importance, targeting these variables in the inspection phase of bridge assessment results in a more stable model from which β can be determined.

The effect on β of modelling these variables as deterministic parameters rather than stochastic variables can be determined from the omission sensitivity factor γ_i, which is a function of α_i^2 (Eqn. 2.23). This factor ratio of the β' and β shows the relative error of replacing a stochastic variable with a deterministic parameter a probabilistic assessment (Table 3.4), and in general, shows little correlation between the variables X_{ij} when comparing variable or uniform variation. However, the relative ranking of these factors remained similar, with small relative movement typically; with the exception of X_{i6} and X_{i7}, which experienced jumps of from highest ranking to lowest, and lowest to highest between variable and uniform variation models. This can be explained due to their CoV values being both the smallest and largest in the initial model with variable CoV, and thus under the uniform CoV of 0.1, they experienced the greatest relative change in their stochastic importance. This further demonstrates the sensitivity of a reliability model due to initial modelling of its stochastic parameters, and highlights the importance of which disparate information levels and uncertainty have on a probabilistic model.

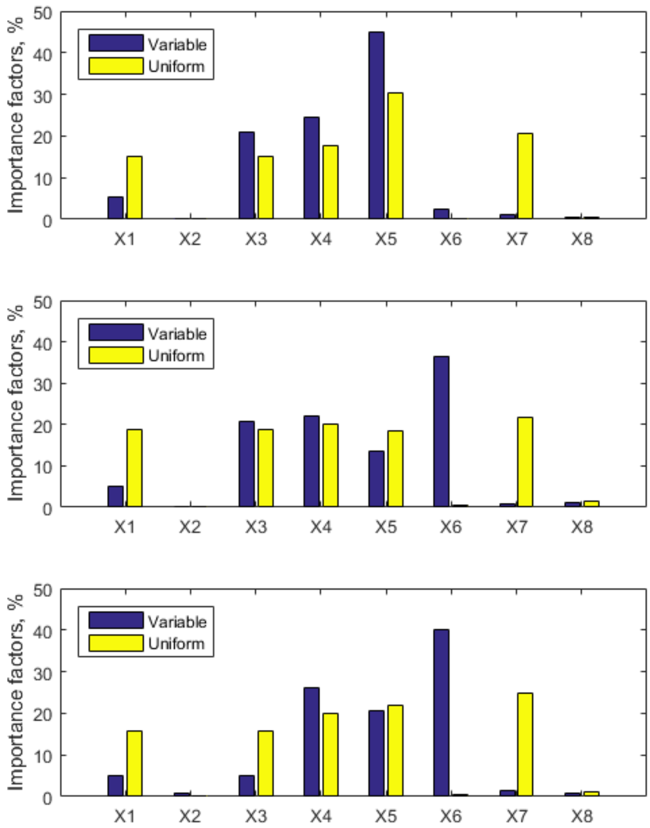

Figure 3.7: Importance factors for probabilistic variables under variable and uniform uncertainty.

Table 3.4: Omission sensitivity factors for variable and uniform CoV

	Slab		Beam		Prestressed	
Tag	Variable	Uniform	Variable	Uniform	Variable	Uniform
X_{i1}	1.027	1.086	1.027	1.110	1.026	1.069
X_{i2}	1.001	1.001	1.000	1.000	1.004	1.001
X_{i3}	1.125	1.086	1.123	1.110	1.026	1.069
X_{i4}	1.150	1.103	1.133	1.119	1.163	1.090
X_{i5}	1.347	1.197	1.075	1.108	1.122	1.065
X_{i6}	1.012	1.001	1.255	1.003	1.292	1.002
X_{i7}	1.006	1.123	1.005	1.129	1.007	1.115
X_{i8}	1.003	1.003	1.006	1.008	1.005	1.173

3.5 Conclusions

A structural reliability analysis was conducted on three bridges of typical construction type in Ireland and mainland Europe. These bridges were assessed for the flexural limit-state under a probabilistic load model, considering various levels of uncertainty in the modelled random variables. The computed estimates of the reliability indices were presented, along with associated parametric sensitivity and importance measures. It was observed that bridges of similar structural arrangement and form are clustered in terms of sensitivity or parametric importance studies. The levels of correlation evident in the parameters when considering different degrees of uncertainty indicates the potential for a calibrated baseline model from which further assessments can be carried out, and benchmarks can be obtained relative to the level of uncertainty present in the model. This calibration would be strongly dependent on the availability and the quality of information of the bridges within a network. This emphasizes the need for data-sharing for such structures by the managers and owners of bridge networks for the most reasonable and cost-effective interventions to be carried out. Further work is encouraged on a wider range of bridges under improved probabilistic information, in order to establish if baseline safety classifications can be established for further specific bridge types. While this chapter was primarily concerned with the uncertainty in the resistance variables in achieving research Objective 1, the following chapter will explore the effects of changing interpretation of load models and their effect on safety classifications of concrete bridges.

Chapter 4

Reliability Analysis under Traffic Loading Uncertainty

4.1 Introduction

4.1.1 Overview

While the previous chapter was mostly concerned with the resource or capacity variables R in the reliability analysis of existing bridges, this chapter will be concerned with the effect of uncertainty in information on applied loading due to the effect of changes in the definitions of the demand variable S (Equation 1.1), through the evolving nature of traffic loading definitions in codes and through newer, probabilistically defined loading models.

With the continued evolution of traffic live loading specifications, safety classifications of bridge structures are subject to change, independent of the actual condition of the structures at that point in time. As investment decisions are often based on these safety classifications, a reclassification of safety level due to changing of live load definitions can lead to misinterpretation of the actual state of the structure, and thus lead to a misallocation of resources. On the other hand, should a reclassification of safety occur after a change in live load specification, the question as to whether modern design traffic loading leads to more or less robust bridges than previous design codes is raised. To investigate this, the same three bridges used in Chapter 3 were assessed for evolving definitions of live load. Using deterministic and probabilistic methods, critical limit-states were assessed and the associated reliability indices and parametric sensitivity factors were determined and compared across various code specifications. This comparison allowed for the evaluation as to how the evolution of live load over time influences the computed safety of bridge structures. Additionally, this chapter investigates the effects that increased design traffic loading have on the initial construction cost and whether that could be balanced by a reduced requirement for financial intervention in the mid to later stages of the bridges design-life. This is investigated by conducting a life-cycle performance and cost assessment on a reinforced concrete slab bridge that is designed to increasing standard traffic loads.

4.1.2 Background

Preserving a functional and serviceable civil infrastructure network requires complex methods to devise optimum strategies to schedule expensive preventative and essential maintenance of existing bridge stock (Estes and Frangopol 2001b). Quantification of structural safety and redundancy for bridges is an important process in network maintenance management (Akgül and Frangopol 2003, Frangopol and Nakib 1991, Weninger-Vycudil et al. 2015) and is strongly dependent on the effects of live loading (Nowak et al. 1993, Nowak 1993). Markers of quantification have evolved from basic definitions of allowable stress indices, to limit-state design, and, eventually, to fully probabilistic reliability analysis (Ellingwood 1996, O'Connor and Enevoldsen 2007, Dawe 2003). While new bridge structures conform to and benefit from the acknowledgement of epistemic and aleatory uncertainties (Ang and Tang 2007) through normative documents (Cornell 1969, Benjamin and Lind 1969, Shah 1969, Lind 1972, Rosenblueth and Esteva 1972), much of the global bridge stock originate from a time when the design of structures was based on basic models and engineering judgement. The nature of these bridges has not fundamentally changed over time, except for the consideration of degradation. Yet, there has not been sufficient funds for owners of bridge stock to replace, intervene, or even prioritise investment (Ellingwood 2005, Frangopol and Liu 2007b, Mueller and Stewart 2011, Frangopol 2011, Frangopol and Soliman 2016, Pakrashi et al. 2011, Frangopol and Bocchini 2012).

Performance indicators are used as a significant decision tool when evaluating intervention options when structural safety is of primary concern. Even after considering a full probabilistic regime, it is important to assess how the markers of safety, expressed as a reliability index β or other performance indices, have changed over time with changing benchmarks of live loading. The evolution of such indices over time, combined with degradation patterns and maintenance intervention is yet to be investigated. Site-specific live loading, related to extreme value distributions fitted to assumed or observed data, through weigh-in-motion (WIM) technology, has shown to have significant potential for assessing the effects of live loading (O'Connor et al. 2001, O'Connor and O'Brien 2005, Caprani and O'Brien 2010, O'Brien et al. 2015a,b). However, too often is the performance of bridges within a network, and thus economic decisions made regarding intervention options, determined using generalised normative/code-based descriptions of traffic loading that are subject to change over time. The use of such methods can thus misinform bridge managers and stakeholders by significantly underestimating the true performance measure of the bridges within their networks.

In most European countries, the basis of assessment calculations is the same as for the design of new bridges (Zonta et al. 2007). At the design stage, traffic loading is typically specified in codes of practice as a function of bridge geometry, with section capacities being designed accordingly (Dawe 2003). In practice, the structural capacity is minimised at the ultimate limit state in an effort to reduce material quantities and initial construction costs. Achieving such economy is often accomplished at the expense of structural robustness from a life-cycle perspective. This chapter investigates to what extent design traffic loading has on life-cycle safety and cost assessments for bridge structures. As deterministic methods, such as LRFD, are predominantly used in structural capacity assessment across a network of bridges, these changes in definitions can have significant impacts on resulting intervention decisions and resource allocation. In this chapter, a brief history of the major bridge design and assessment standards will be presented, and the effect of the varying definitions of code-defined traffic loading will be shown on the performance indicators, in this case the reliability index β (Ditlevsen and Madsen 1996, Melchers 1999, Pakrashi and Hanley 2015), of three simply supported concrete bridges of the same span. These changes will be benchmarked against β from site-specific traffic loading, and the effect changing code-defined traffic loading has on the probabilistic model will be shown through parametric sensitivities and importance factors (Madsen et al. 1986). The type of bridges used in this assessment were chosen based on their high level presence within Europe and the UK (Žnidarič et al. 2011). An 80 year reliability assessment is also presented, showing how β

can transition below a minimum acceptable threshold at a single point-in-time due to code changes coupled with typical degradation effects, as well as an associated life-cycle cost assessment that is required to keep the bridge above a minimum acceptable performance threshold, defined as the target reliability index β_T. It is shown how a small relative increase in the flexural capacity at the design stage, and thus initial construction cost, results in a significant offset in the expect cost of failure, and thus the total expected life-cycle cost.

4.2 Evolution of Normative Live Loading

Prior to the latter 19th century, traffic loading on bridges was not of primary concern to the bridge builder, as this load was considered light relative to the self-weight of the structure itself (Henderson 1954). It was only subsequently due to the emergence of the traction engine that the effect of traffic loading on bridges became an important design criteria. The evolution of normative traffic load specifications in the UK and Ireland, from the suggestion of nominal wheel loads to a standard loading curve (SLC), is detailed at length by Dawe (2003) and is summarised in Table 4.1. While many minor changes to these normative documents have been made in the past century, the five major changes will be discussed in this chapter; *BS 153* (BSI 1937), *BS 5400* (BSI 1978), *BD 21/84* (Highways Agency 1984), *BD 37/88* (Highways Agency 1988), and the introduction of the *Eurocode* (CEN 1994).

4.2.1 BS 153

BS 153–Standard specification for girder bridges (BSI 1937) was developed by the British Standard Institution (BSI) in 1937 for the design and construction of girder bridges, *part 3* of which dealt with the application of traffic loading. The standard recommended the use of a standard loading train (SLT) with a unit load of 1 ton/axle, and 15 units to be applied per 10 ft of lane width, and a 10 ft headway between vehicles. Additionally, it was specified to apply a uniformly distributed load (UDL) of 4.02 kN/m^2 (84 lb/ft^2) to account for pedestrians and light traffic. Further revisions of this standard introduced what is now known as 'abnormal' loading, with the previous loading being referred to as 'normal' loading, as well as the increase in applied units from 15 to 22 to account for general traffic increases. Furthermore, computational ease was improved with the introduction of a standard loading curve (SLC) to replace the SLT model. The SLC specified a UDL as a function of span, with a higher UDL for shorter spans to account for the increased likelihood of a single span being fully loaded by trucks. Additionally, a knife-edge load was to be applied across the lane width of 39.4 kN/m (2700 lb/ft) at a location within the span to produce the worst shear force effect.

4.2.2 BS 5400

The introduction of *BS 5400–Steel, concrete, and composite bridges* (BSI 1978) in 1978 transitioned standards to the limit-state philosophy, whereby partial factors could be applied to both load and resistance variables (Allen 1975). *Part 2* of the standard dealt with the application of traffic loads, and recommended a 5% characteristic value for the ultimate traffic load; having a 5% chance of occurring within the design life of the structure, set as 100 years. The limit-state philosophy is designed to allow for the benefit of statistical knowledge to more accurately model expected scenarios. However, at the introduction of *BS 5400*, such data was not available, and so nominal loading and partial factors were specified, based on engineering judgement at the time. The SLC from *BS 153* was retained, except with a constant UDL of 30 kN/m/lane up to a span of 30 m. For simply supported spans, this resulted in a maximum midspan bending moment slightly less than that prescribed in *BS 153*, for which a divergence begins from the 30–50 m span range (Figure 4.1).

4.2.3 BD 21/84

BD 21–The assessment of highway bridges and structures (Highways Agency 1984) was introduced in 1984 revise some provisions of *BS 5400* for shorter spans. Specifically, the furthest departure was the elimination of a constant UDL for spans under 30 m, to be replaced by a curve that was fully variant with span length, and defined by a single formula as a function of length. The apparent lifetime of a bridge was extended to 120 years, so whereby a 5% characteristic ultimate load over the design life resulted in a total return period for the ultimate load of 2,400 years. The development of this code involved a more rigorous calibration of partial factors using statistical methods than the previous standard employed. The SLC was developed under the assumption that

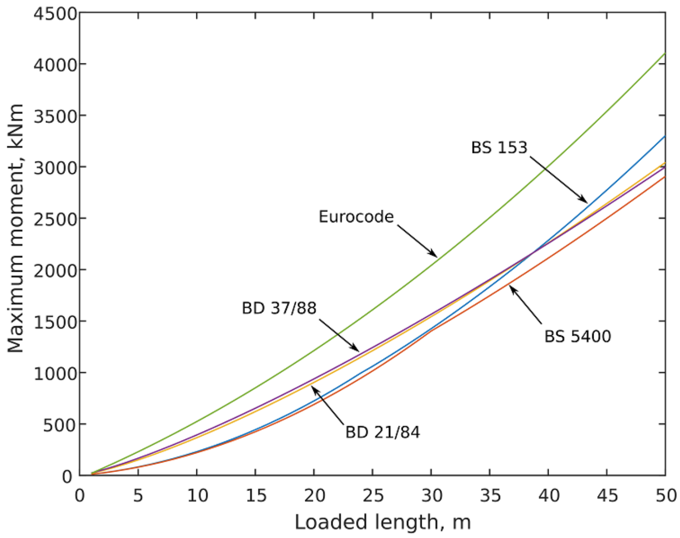

Figure 4.1: Maximum bending moment with increasing spans for changing live load definitions

shorter spans are more likely to be fully laden with convoys of large vehicles than larger spans, and thus envelopes were made of the worst load effects for a variety of spans, and a new single SLC was derived from the results. The effect of the elimination of a constant UDL for spans under 30 m can be seen through the deviation between maximum bending moments for *BS 5400* and *BD 21/84* in Figure 4.1.

4.2.4 BD 37/88

Due to the general expected increase in total weight of European vehicles, the SLC of *BD 21/84* was updated in *BD 37–Loads for highway bridges* (Highways Agency 1988) to account for a 40 tonne gross weight vehicle, as opposed to that of *BD 21/84* which accounted for 38 tonnes. This code also featured a 'composite' version of *BS 5400*, which included specifications for railway loading. The effect of this code is scene in greater prominence for spans above 50 m, but produces a minimal change in flexural load effects from *BD 21/84* (Figure 4.1).

4.2.5 Eurocode

The development of *EN 1991-2: Eurocode 1: Actions on structures. Traffic loads on bridges* (CEN 1994) introduced four separate load models to account for the vertical load being applied to bridges, with Load Model 1 (LM1) corresponding to what has been referred to as normal loading, for spans between 5–200 m, and a carriageway width of up to 42 m. LM1 was derived from real European traffic data, and specified an ultimate load exceedence rate of 5% in 50 years, or a return period of 1000 years (Bruls et al. 1996). LM1 departed from previous representations of normal traffic loading by eliminating the SLC defined UDL and invariant KEL, and replacing them with a series of constant UDL, invariant with span length, in adjacent lanes and a tandem axle system of point loads.

Table 4.1: Development of traffic loading rules, abridged from Dawe (2003)

Date	Event/publication	Comment
End of 19th century		Principal live loading on bridges deemed to be due to crowd loading. UDL used for design of bridge decks, for example 4.8 kN/m^2 for Hungerford Suspension Bridge
1904	Restriction on vehicle weights	8 ton limit for single axle, 12 ton limit for gross vehicle weight
1923	BS 153 *Part 3: Loads and stresses*	Traffic live loading to be specified by the Engineer. Impact factor inversely proportional to span.
1931	MoT *Standard loading for highway bridges*	Standard Loading Curve. Deterministic approach using equivalent UDL and KEL, with allowance for impact. Heavy wheel load introduced for short span structures.
1937	BS 153 Part 3 (1st revision)	Introduced Types A and B loading. Impact allowance varied with span
1954	BS 153: Part 3A (2nd revision)	Appendix A introduces Types HA and HB loading. HA comprises deterministic formula loading based on 22-ton vehicles, and an alternative wheel loading. HB loading with axle number and spacing based on typical abnormal trailers of the day; axle loads are heaviest allowed by law. (metricated in 1972)
1973	DoE technical memorandum (bridges) BE 5/73, *Standard highway loadings*	Loads applicable to all highway structures except steel box girders. Required a minimum of 30 units of HB loading for public roads. HA UDL capped at 31.5 kN for loaded lengths up to 6.5 m. HA wheel load and HB loading assumed to cover design of short spans.
1978	BS 5400 Part 2, *Specification for loads*	Introduction of limit state design. HA loading based on 24-tonne vehicles. HA UDL capped at 30 kN/m for loading lengths up to 30 m. Minimum UDL intensity now required to be 9 kN/m. Minimum of 25 units of HB required for public roads. HB loading (and HA wheel load) assumed to cover design of short spans.
1982	DTp BD 14, *Loads for highway bridges*	Implemented BS 5400: Part 2 for loaded lengths up to 40 m.
1984	DTp BD 21, *The assessment of highway bridges and structures*	HA loading re-derived for Construction and Use vehicles, taking into account effects of overloading, lateral bunching and impact factor of 1.8. Loading derived for a full range of spans (i.e. no longer capped for short spans).
1988	DTp BD 37, Loads for highway bridges (composite version of BS 5400: Part 2). Incorporated in DMRB in 2001	Revision of BS 5400: Part 2: 1972 containing revised HA loading; short span based on BD 21/84, enhanced long span derived statistically from live traffic data. Covers spans up to 1600 m.
1994	CEN, ENV 1991-3. *Eurocode 1: Basis of design and actions on structures. Part 3: Traffic loads on bridges*	European pre-standard for traffic loads on bridges. Covers spans up to 200 m. Constant UDL for all spans and tandem axle systems. 3 m notional lanes. (Issued in 2000 together with UK NAD. Constant UDL for all lanes across carriageway.)
1997	HA, BD 21. The assessment of highway bridges and structures. Revised in 2001	Revision of short span assessment loading by statistical methods and allowing for site factors for volumes of traffic and road surface condition.

4.2.6 Summary

As can be seen from the comparison of bending moments in Figure 4.1, LM1 of *Eurocode* results in the most onerous of load effects of the presented normative standards. This can further be seen in Figure 4.2 where the *Eurocode* loading can produce a bending moment of as much as three times that of the initial bending moment M_0 due to *BS 153* in a simply support span. This ratio can be expressed as M_x/M_0, where M_x is a bending moment produced by a subsequent normative standard x, and reduces to 2 at 13 m under *Eurocode* and 1.25 at 50 m. For the 16 m spans considered in this and the previous chapter, the ratios of M_x/M_0 for *BS 5400*, *BD 21/84*, *BD 37/88*, and *Eurocode* are 0.96, 1.36, 1.43, and 1.86, respectively. Throughout this change in live load definitions, there has been no substantive change in the capacity models or in the partial factors for materials γ_m; for which the concrete factor $\gamma_{m,c}$ for the ultimate limit-state is 1.5 and the reinforcing/prestressing steel factor $\gamma_{m,s}$ is 1.15, for both the *British Standards* (BSI 1984, 1990) and *Eurocode* (CEN 2004). From this, it can be argued that these later design standards faced larger design requirements than initially for *BS 153*, and so these bridges were designed to provide a larger degree of strength than before; which would typically result in a higher construction cost due to increased use of structural materials, not accounting for variations in raw material cost over time. This, however, does pose the question as to whether this increased initial cost can be off-set by reduced repair costs in stronger bridges throughout their life-cycle.

4.3 Assessment Model

A reliability analysis was carried out under the flexural limit-state as per the previous chapter. However, in addition to the analysis under the probabilistically defined load model used previously; in this chapter a comparison is being sought between various deterministic load models defined in codes of practice. While these models have not been typically defined using statistical methods, in order to draw a direct comparison between the load models and the effects that their changes may have on structural parameters, such as importance factors or parametric sensitivity, a notional reliability analysis in conducted under each load model. If real information around traffic loading was available over time, then it would have been possible to create a traffic load distribution, including

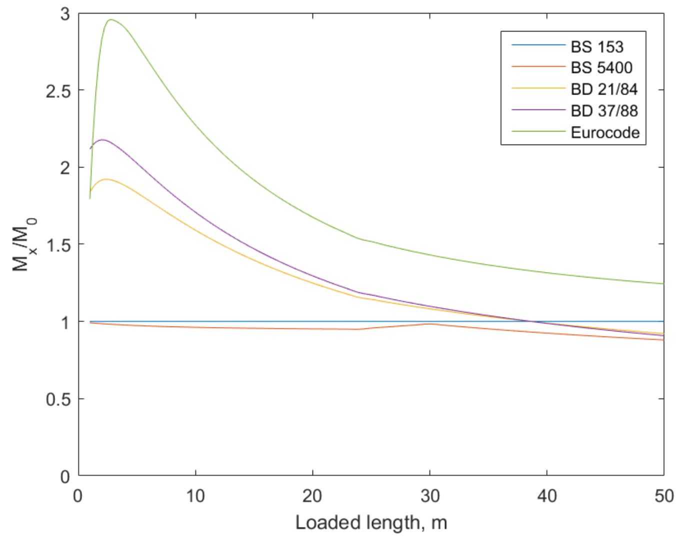

Figure 4.2: Maximum bending moment ratio M_x/M_0 with increasing spans for changing live load definitions

possible evolution of it in time. However, in the absence of such a model, normative assessment uses the traffic load definition from codes. It is often difficult to translate these definitions into an equivalent probabilistic distribution and a related value.

However, should there only be definitions from codes available, it is expected that any such equivalent value will reflect the changes in definitions from codes; and reliability indices computed from that limited level of information will reflect changes in definitions in codes. While such historical evolution provides a commentary on how normative safety indices have changed over time, adopting a probabilistic study, based on the assumption that limited and only code-based traffic loading definitions are present, will allow for investigation into what codes provided what levels of estimated safety; along with information around parametric sensitivity and parameter importance measures. To reasonably conduct this, the undamaged reliability index assessed from this analysis is benchmarked against known values for bridges under the probabilistic traffic load model; so that this method of investigation does not represent significantly unrealistic scenarios and the comparisons are carried out with examples with a realistic degree of confidence.

In order to represent the load in a probabilistic space, and with reference to equations 3.2, 3.3, and 3.4, the random variable λ_{Prob} is replaced with a live load uncertainty factor λ_{LL}, which is modelled as a lognormal distribution with values (μ, σ) of (1.0,0.2) (Akgül and Frangopol 2004b). Additionally, the deterministic coefficients C_{05}, C_{15}, and C_{26} can now be expressed as:

$$C_{ij,a} = \left(\frac{32.106L^2}{8} + \frac{120L}{4}\right)\frac{b}{1000b_L}$$

$$C_{ij,b} = \left(\frac{30L^2}{8} + \frac{120L}{4}\right)\frac{b}{1000b_L}$$

$$C_{ij,c} = \left[\frac{260\left(\frac{1}{L}\right)^{0.6}L^2}{8} + \frac{120L}{4}\right]\frac{b}{1000b_L}$$

$$C_{ij,d} = \left[\frac{336\left(\frac{1}{L}\right)^{0.67}L^2}{8} + \frac{120L}{4}\right]\frac{b}{1000b_L}$$

$$C_{ij,e} = \left\{\frac{16.764L^2}{8} + \left(\frac{300\left[\left(\frac{L}{2}+0.3\right)+\left(\frac{L}{2}-0.9\right)\right]}{L}\right)\left(\frac{L}{2}-0.3\right)\right\}\frac{b}{1000b_L}$$

where the subscripts a, b, c, d, and e represent *BS 153*, *BS 5400*, *BD 21/84*, *BD 37/88*, and *Eurocode*, respectively; and the variable b in the equations is the width of section considered for the slab bridge, and the effective flange width b_{eff} for the beam and prestressed concreted bridge (Table 3.3).

At this point it is useful to consider the effects of deterioration over the life-cycle of the bridges in order to determine a time-dependent reliability index $\beta\left(t\right)$. The time-dependent reliability model extends the fundamental model previously discussed by introducing the time function t:

$$P_f(t) = P[R(t) \leq S(t)] \tag{4.1}$$

Where $R(t)$ is the time-varying resistance and $S(t)$ is the time-varying load. With the absence of any future traffic loading, $S(t)$ will be treated as S as per the initial assessment, but $R(t)$ will be evaluated as the loss of capacity over time due to corrosion effects. The corrosion model used in the lifetime assessment of the bridges was based on a uniform reduction in flexural steel area, assumed here to be caused by chloride only (Akgül and Frangopol 2005a). The time to initiation of corrosion T_i is commonly obtained using Fick's 2nd law of diffusion (Akgül and Frangopol 2004c, 2005b, Kenshel and O'Connor 2009):

$$T_i = \frac{C^2}{4D_c}\left[\mathrm{erf}^{-1}\left(\frac{C_s - C_{cr}}{C_s}\right)\right]^{-2} \tag{4.2}$$

where C is the concrete cover to flexural reinforcement (mm); C_{cr} is the critical chloride concentration (%); C_s is the surface chloride concentration (%); D_c is the chloride diffusion coefficient (mm^2/year); and erf is the error function. In this analysis, C_{cr}, C_s, and D_c are treated as random variables with a lognormal distribution; with values (μ, σ) of (0.037,0.0056), (0.15,0.015), and (110,12.1), respectively (Enright and Frangopol 1998). While this represents only one method for modelling corrosion, its use is adequate here for establishing a comparative discussion across the various traffic load models. Once the time to corrosion initiation is determined, time-variant flexural steel $A_s(t)$ area can be found as:

$$A_s(t) = \frac{\pi}{4}\sum_{j=1}^{n}\left[D_{0,j} - \Delta D_j(t)\right]^2, \quad \Delta D_j(t) = r_{corr}\left(t - T_i\right) \tag{4.3}$$

where $D_{0,j}$ is the initial diameter of the steel bars and strands; $\Delta D_j(t)$ is the amount of section lost after time t; n is the number of bars; and r_{corr} is the rate of corrosion of the flexural steel. While r_{corr} is a function of the constant rate in time i_{corr} and the corrosion coefficient value C_{corr}, here r_{corr} (mm/year) is modelled as random variable with a lognormal distribution, with a mean μ and standard deviation σ of 0.0762 and 0.0223 for the RC bridges (Akgül and Frangopol 2005b), and 0.0571 and 0.017 for the PC bridge (Akgül and Frangopol 2004c).

In addition to uniform corrosion, it is acknowledged that localised corrosion can reach a penetration of four to eight times the penetration estimated under the uniform corrosion model (González et al. 1995). This additional penetration can be modelled using the pitting corrosion model (Val and Melchers 1997), where the net cross-sectional area of the steel $A_s(t)$ can be found as:

$$A_s(t) = \begin{cases} \frac{\pi D_o^2}{4} - A_1 - A_2, & p\left(t\right) \leq \frac{\sqrt{2}}{2}D_o \\ A_1 - A_2, & \frac{\sqrt{2}}{2}D_o < p\left(t\right) \leq D_o \\ 0, & p\left(t\right) > D_o \end{cases} \tag{4.4}$$

where A_1, A_2, a, θ_1, and θ_2 are physical parameters of the pit (Figure 4.3), defined as:

$$A_1 = \frac{1}{2}\left[\theta_1\left(\frac{D_o}{2}\right)^2 - a\left|\frac{D_o}{2} - \frac{p\left(t\right)^2}{D_o}\right|\right] \tag{4.5}$$

$$A_2 = \frac{1}{2}\left[\theta_2 p\left(t\right)^2 - a\frac{p\left(t\right)^2}{D_o}\right] \tag{4.6}$$

$$a = 2p\left(t\right)\sqrt{1 - \left[\frac{p\left(t\right)}{D_o}\right]^2} \tag{4.7}$$

$$\theta_1 = 2\arcsin\left(\frac{2a}{D_o}\right) \tag{4.8}$$

$$\theta_2 = 2\arcsin\left[\frac{a}{p\left(t\right)}\right] \tag{4.9}$$

and $p(t)$ is the radius of the pit at time t, being a function of $\Delta D_j(t)$ from the uniform corrosion model (Equation 4.3):

$$p(t) = \Delta D_j(t)R \tag{4.10}$$

where R is a coefficient of the ratio between the maximum and uniform corrosion penetration, and typically has a value of between 4 and 8 which accounts for the difference as noted by González et al. (1995), as specified earlier.

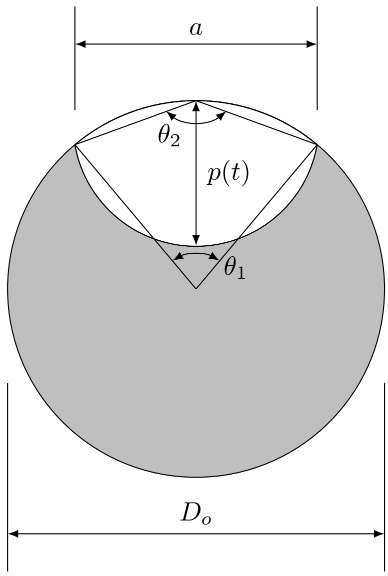

Figure 4.3: Pitting corrosion, adapted from Val and Melchers (1997)

While the use of a pitting corrosion model produces a more critical deterioration scenario when spatial variability is considered (Stewart 2004), the uniform corrosion model will be used in this analysis due to its simplicity in application and where the consideration of time dependent effects are considered from a nominal basis, from which only direct comparison will be made across the results. While the use of the pitting corrosion model would result in variation in the results than those presented, and would be expected to be less favourable, the variations would be relative across the bridges and so the conclusions drawn would be the same as the presented assessment using the uniform corrosion model.

4.4 Life-Cycle Cost Model

The life-cycle cost model used in this assessment was developed by Frangopol et al. (1997) to optimise the inspection and repair of deteriorating structures, the procedure of which is briefly summarised here. The expected total life-cycle cost C_{ET} is the sum of the various cost components of the structure; initial construction C_T, routine preventative maintenance C_{PM}, inspections C_{INS}, repair C_{REP}, and failure C_F.

$$C_{ET} = C_T + C_{PM} + C_{INS} + C_{REP} + C_F \tag{4.11}$$

The initial construction cost C_T is taken as a function of the volume of concrete and steel in the section, and can be expressed as:

$$C_T = C_c A_c L + C_s A_s L \tag{4.12}$$

where C_c and C_s is the unit cost of concrete and steel per m^3, respectively; A_c and A_s is the area of concrete and steel in the section, respectively; and L is the length of the section being considered. For this analysis, C_c was chosen to be €36/m^3 and C_s was chosen to be €1785/m^3; for a C_s/C_c ratio of approximately 50:1 (Lin and Frangopol 1996). In order to account for the reduction of A_s away from the position of maximum bending moment, it is suggested by Lin and Frangopol (1996) to factor A_s by 0.75. However, as this model does not consider the effect and cost of shear reinforcement, the total value of A_s will be accounted for to simulate the cost of shear reinforcement.

The cost of lifetime preventative maintenance C_{PM} is described as the linear combination of the cost of preventative maintenance at year one C_{main}, and the age of the structure at the time of the preventative maintenance t. To account for future costs, C_{PM} is the sum of the net present value costs of each occurrence of routine preventative maintenance:

$$C_{PM} = \sum_{i=1}^{t} C_{main,i} \frac{1}{(1+r)^{t_i}} \tag{4.13}$$

where r is the net discount rate.

The total expected inspection cost C_{INS} is defined as:

$$C_{INS} = \sum_{i=1}^{m} C_{ins} \frac{1}{(1+r)^{t_i}} \tag{4.14}$$

where m is the number of inspections; and C_{ins} is the cost of the inspection method used (Mori and Ellingwood 1994), which is a function of the detectable damage intensity η and cost of an ideal inspection α_{ins}, which is to be taken as a fraction of the initial cost C_T.

In the development of this model, Frangopol et al. (1997) have shown that the number of inspections m have a significant influence on C_{ET}, as this variable has a direct influence on the number of repair activities carried out, which reduces the overall probability of failure P_f and thus the expected cost of failure C_F. To establish C_{REP} and C_F, an event tree can be constructed whereby for each inspection, a decision can be taken on whether to initiate a repair activity or not. A constraint imposed on the model is that if damage η is detected during any inspection, a repair activity must be carried out. This decision is thus based on the probability of detecting damage at the time of an inspection, or the probability of damage not being detected. For simplicity, in this assessment it is assumed that each repair activity returns the bridge to its initial reliability index β_i. Thus, for each node of the event tree, the decision on whether to conduct a repair activity or not will directly influence the failure probability at the following node on the event tree for the next inspection.

The lifetime failure probability $P_{f,life}$ of the bridge for any number of m inspections can be defined as:

$$P_{f,life} = \sum_{i=1}^{2^m} P_{f,life,i} P(B_i) \tag{4.15}$$

where $P(B_i)$ is the probability that any path on the event tree occurs (Figure 4.4), and $P_{f,life,i}$ is the maximum failure probability for that path. Each branch B_i represents a specific sequence of repair events b_i^j, which can be defined as repair occurring or not occurring after an inspection; the probability of which is determined as a function of the damage intensity $\eta\,(t)$ at the time of inspection and the probability of detecting this damage $d\,(\eta)$. These can be defined using the following expressions:

$$\eta\left(t\right) = \begin{cases} 0, & 0 \leq t \leq T_i \\ \frac{D_{b0} - D_b(t)}{D_{b0}}, & T_i < t \end{cases} \tag{4.16}$$

$$d\left(\eta\right) = P\left(\text{damage detection} \mid \eta\right) = \begin{cases} 0, & 0 \leq \eta \leq \eta_{min} \\ \Phi\left(\frac{\eta - \eta_{0.5}}{\sigma}\right), & \eta_{min} < \eta \leq \eta_{max} \\ 1, & \eta > \eta_{max} \end{cases} \tag{4.17}$$

Where D_{b0} is the initial bar diameter; $D_b\left(t\right)$ is the time-dependent bar diameter based on the corrosion model; $eta_{0.5}$ is the damage intensity at which the NDE method has a 50% probability of detection; σ is the standard deviation; and η_{min} and η_{max} are the minimum detectable damage intensity and the damage intensity for which probability detection is certain, respectively.

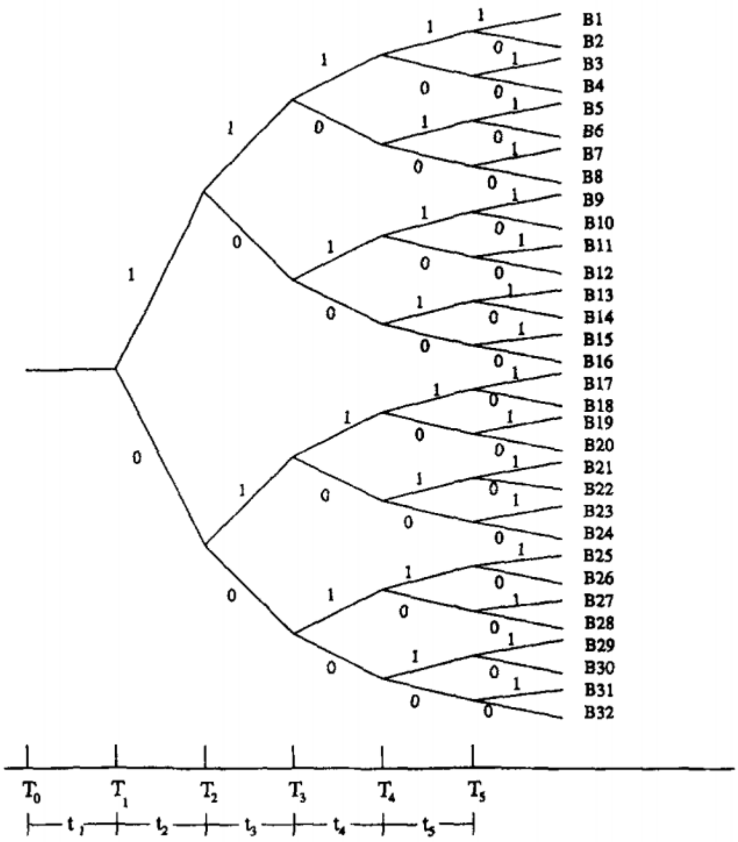

Figure 4.4: Event tree of repair path for five inspections (Frangopol et al. 1997)

The expected total cost of repair is then defined as:

$$C_{REP} = \sum_{i=1}^{2^m} C_{rep,i} P(B_i) \tag{4.18}$$

where $C_{rep,i}$ is the net present value repair cost at each node, as a function of the effect of the repair activity e_{rep}, defined as:

$$e_{rep} = \frac{\bar{M}_{r,a} - \bar{M}_{r,b}}{\bar{M}_{r0}} \tag{4.19}$$

Where $\bar{M}_{r0}$ is the original mean flexural capacity; and $\bar{M}_{r,b}$ and $\bar{M}_{r,a}$ are the mean flexural capacities before and after repair, respectively.

Finally, the expected failure cost C_F is defined as:

$$C_F = C_f P_{f,life} \tag{4.20}$$

where the failure cost C_f is a function of the initial cost C_T; in this assessment assumed to be 1,400C_c, for illustrative purposes. For a full life-cycle cost analysis, this failure cost would need to be specified with greater detail, and account for economic and social consequences of failure. The model used here is so as to benchmark the results against those obtained in the formulation of this life-cycle cost model by Frangopol et al. (1997). By using this model, it will be possible to offer nominal comparisons between the expected life-cycle cost of bridges designed under increasing design traffic loading, and show whether this increase in capacity requirements, thus increased initial construction costs, can be off-set by reduced necessity for remedial intervention throughout the bridges lifetime. While other models exist for estimating life-cycle costs of civil infrastructure (Cho 2009), this method has been deemed adequate to address the objective posed in this chapter.

4.5 Results

4.5.1 Undamaged Reliability Assessment

A reliability assessment was conducted on the three bridges under consideration to determine the relative change in β for each variation in code-defined traffic loading, not considering degradation. These values of β were compared to that for β_{slab}, β_{beam}, and β_{pres} from Chapter 3, which were computed from the probabilistic load model. The ratio of the reliability index for each code defined load model β_x and under the probabilistic load model β_{Prob} can be seen in Figure 4.5; where ratios closer to 1 represent little variation between β assessed under the code-defined or probabilistic load model, and ratios closer to 0 represent large deviation between the models. As can be seen, despite an increase in β from *BS 153* to *BS 5400*, there is a consistent decrease in β with more recent code-defined traffic loading. Additionally, with more recent code-defined loads, the disparity between β for specified loading and the probabilistic load model is increased.

As the return periods for the code-based loading is quite high, this disparity between specified loading and site-specific probabilistic loading is expected; and so with greater disparity, more conservative structures are being designed, and thus the probability of the limit state being violated under regular use is lowered. This, however, can not be said to be the case for *BS 153* to *BS 5400*, which have much closer β's to the probabilistic load model. This would suggest that the load effects produced by the ultimate traffic load in these early codes are actually more representative of that produced by the typical traffic load from the probabilistic model. This is problematic, as these ultimate loads are not expected to occur within the reasonable life-cycle of the bridge structure. The low relative value of β under *Eurocode* is expected given it produces the most adverse bending moment of the presented standards (Figure 4.1). However, the discrepancy between this β and that for the site-specific loading suggests that it is perhaps too onerous for the purposes of assessment for existing structures, but designing new bridges to this requirement will produce more robust structures.

4.5.2 Parametric Sensitivity & Importance Factors

The importance factors α_i^2 were determined to highlight the random variables that have the greatest influence on β, for each new definition of code-based traffic loading (Figure 4.6). The importance factors which demonstrate the biggest variation are the random variables X_5 and X_8, which correspond to the uncertainty factors for concrete λ_c and live load λ_{LL}. This would suggest a diminishing role of the self-weight of the bridges as the traffic loading becomes more onerous. For RC and PC beam bridges, λ_{LL} has the highest importance

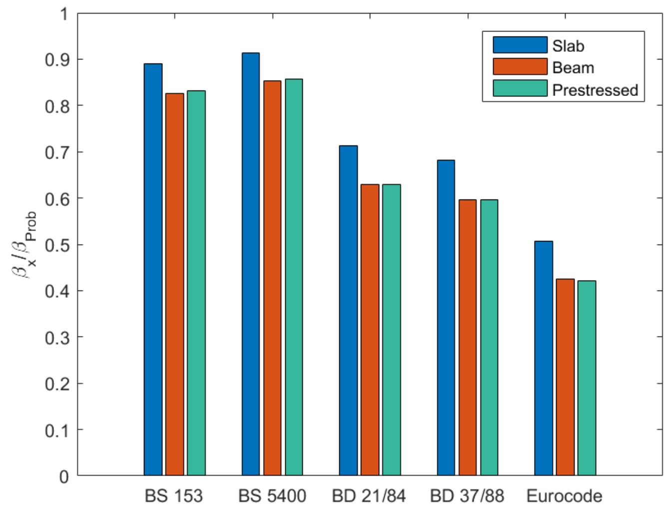

Figure 4.5: Ratio of reliability index for probabilistic load model to code defined load model, not considering structural degradation

factor across all the codes, with a lower bound value of 30.3% and 31.7% for *BS 5400*, and an upper bound value of 45.3% and 51.4% for *Eurocode*, respectively. However, for the RC slab bridge, it can be seen that the importance factors for these random variables occupy the same range throughout the changing codes, except for an almost inverse relationship between the self-weight and the live load. For *BS 5400*, the importance factors for λ_c and λ_{LL} are 34.2% and 11.8%, respectively; whereas, for *Eurocode*, they are 15.6% and 30.7%, respectively. The greater influence of the self-weight is expected for the slab bridge, due to its inherent form of mass concrete, as opposed to the RC and PC beam bridges, which are lighter in nature. It can be seen that the importance factors for each of these variables are somewhat equal for *BD 21/84* and *BD 37/88*, before the more onerous traffic loading of *Eurocode* becomes the most dominant importance factor.

The parametric sensitivity α_i was demonstrated by assessing the effect on β of a 10% perturbation in the mean value of the random variables (Figure 4.7). It is evident that the most favourable random variables across the three bridges are X_1, X_3, X_4, and X_7, corresponding with $A_{s,p}$, $f_{y,pu}$, γ_m, and λ_d. The only random variable which exhibits any significant variation with changing codes is the model uncertainty for flexure γ_m, with the remaining favourable random variables maintaining their relative sensitivities. However, the variation remains only slight, but is indicative of how the code-defined traffic loading becomes more onerous and, thus, more dominant in the probabilistic model. It is note-

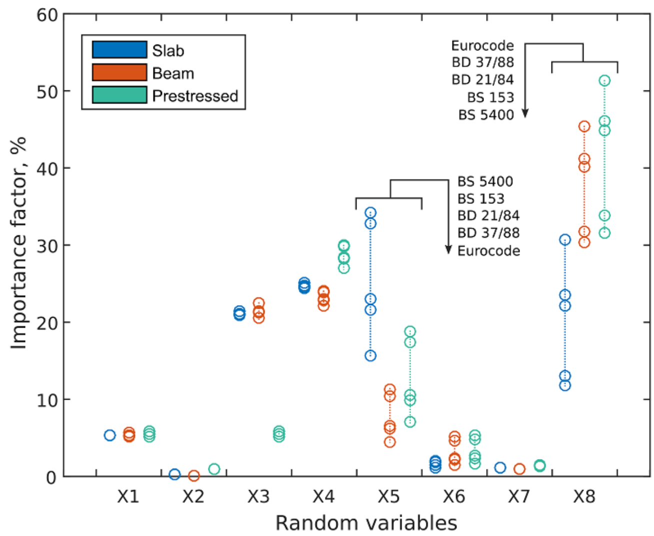

Figure 4.6: Importance factors of the random variables for each code specification

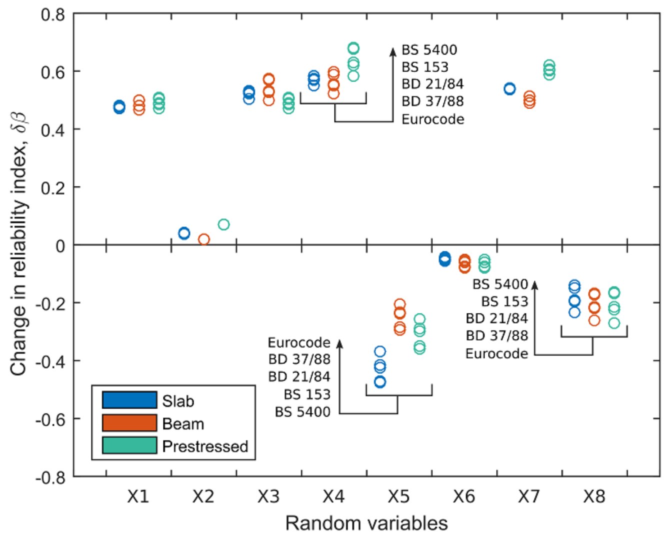

Figure 4.7: Parametric sensitivity of β for a 10% perturbation in the random variables

worthy how, for the PC beam bridge, the grade of prestressing steel f_{pu} has a low importance measure (Figure 4.6), yet is in line with the grade of reinforcing steel f_y for the parametric sensitivity, even when f_y is stochastically more important. This can be attributed to the coefficients of variation (CoV) for the two random variables; with f_{pu} having a lower CoV (5%) than f_y (10%), due to the more controlled nature of manufacturing process of precast PC beams, as opposed to in-situ cast RC slabs and beams .

For the unfavourable random variables, X_5 (λ_c), $X_6(\lambda_s)$, and X_8 (λ_{LL}), it can be seen that the uncertainty factor related to concrete self-weight λ_c displays the greatest negative relative change in β for a 10% perturbation. Additionally, λ_c for the RC slab bridge has the greatest parametric sensitivity, which is consistent with the established importance factors (Figure 4.6). While the sensitivity of λ_c across the code variations remains the highest for the RC slab bridge, it can be seen that the relative ranking of sensitivities is switched between that for λ_c and λ_{LL} for the RC and PC beam bridges. This is more prevalent for the RC beam bridge, where the relative change in β for λ_c and λ_{LL} under *BS 5400* is -0.29 and -0.17, and under *Eurocode* is -0.20 and -0.26, respectively. This shows the same somewhat inverted relationship between these two codes as has already been seen earlier. For the PC beam bridge, these two variables have a relative change in β of -0.36 and -0.16 under *BS 5400*, and then converge to -0.26 and -0.27 under *Eurocode*, respectively.

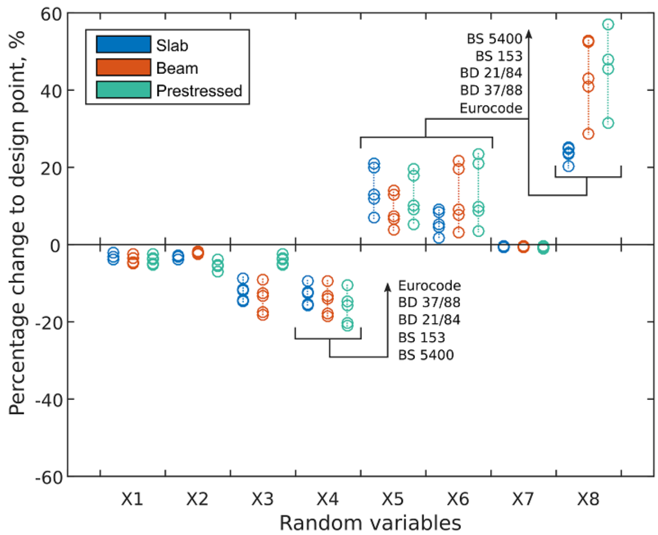

Figure 4.8: Relative change in the random variables at the design point for each code specification

The percentage change in each of the random variables at the design point $\mathbf{u}^*$, being the most likely point of failure, can be seen in Figure 4.8. It is apparent that, under *Eurocode*, the variables require the least amount of deviation from the mean value to reach $\mathbf{u}^*$, whereas for *BS 5400*, the variables require the largest deviation. This variation between the two codes is most pronounced for λ_{LL}, and is consistent with the relationship seen for the importance factors (Figure 4.6). Again, this further emphasises the more onerous nature of the more recent codes, over the earlier models.

4.5.3 Life-Cycle Reliability Assessment

The life-cycle assessment was conducted through a time-dependent reliability analysis, considering the time-variant degradation of flexural steel area due to the uniform corrosion model. The reliability indices computed in this analysis are single-point-in-time, annual figures to demonstrate the expected future β at a specific point in time in the future. Using Equation 4.2, the time to corrosion initiation T_i was evaluated using a Monte Carlo simulation of 50,000 samples, and fitting a lognormal distribution as a good estimate (Enright and Frangopol 1998). The mean value of T_i for both RC bridges was 24.1 years, and for the PC bridge is 15.4 years for the first layer of steel and 51.8 years for the second layer of steel. The loss of cross-sectional area of flexural steel was determined using equation 4.3 and plotted for each bridge over an 80 year period (Figure 4.10).

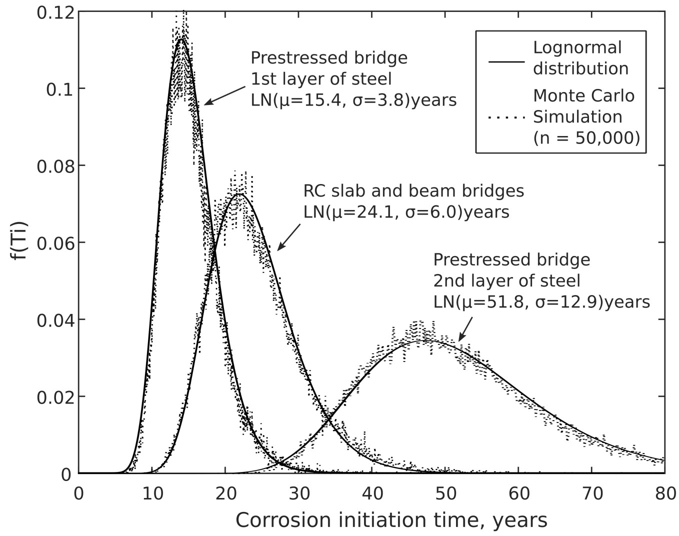

Figure 4.9: Probability density function of corrosion initiation time for each bridge with lognormal distribution and Monte Carlo Simulation

The effect of corrosion on β for the three bridges can be seen in Figure 4.11, where the time-varying reliability is presented for the probabilistic load assessment. Included in Figure 4.11 is a normative safety index based on code-defined loading; including ‘jumps’ in this safety index that account for the changing code specifications over time. For the flexural limit-state, this safety index is defined as the ratio between the maximum moment under traffic loading M_d and the moment capacity of the section M_u; where values in excess of 1 represent failure of the limit-state under a normative LRFD assessment. The ‘jumps’ in

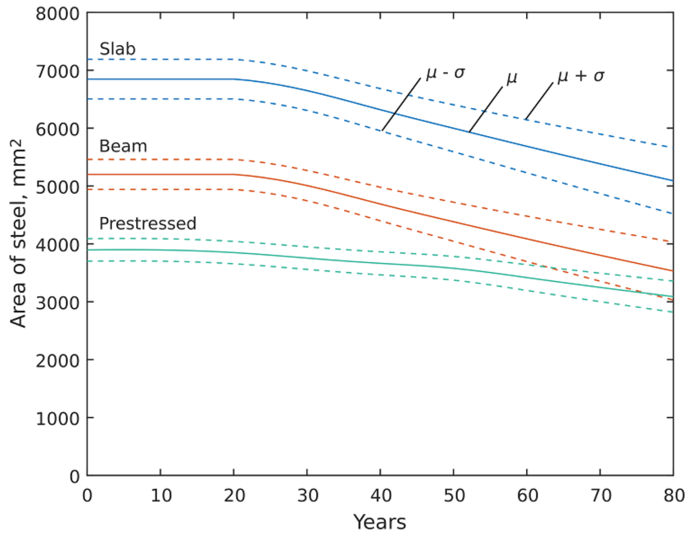

Figure 4.10: Deterioration of steel area on RC and prestressed bridges

safety index represent periods where code-defined traffic load models changed to a newer model, and the rapid change in safety index are thus expected when using only these normative models.

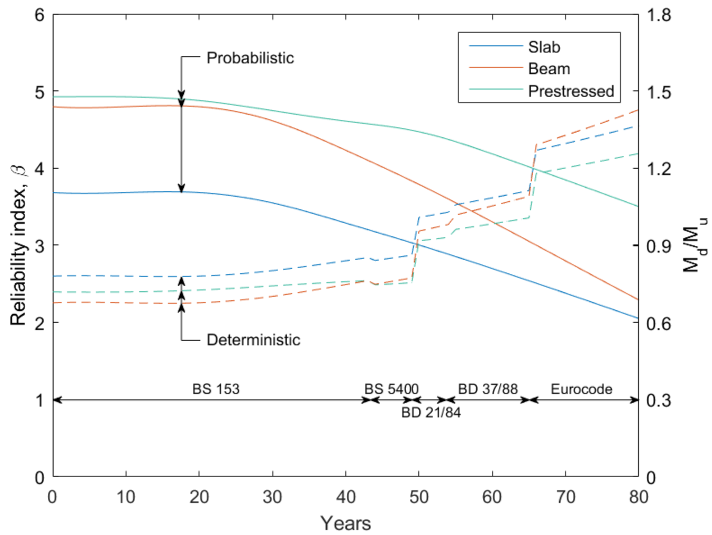

Figure 4.11: Life-cycle reliability index and normative safety index for flexure

For the RC slab bridge, the initial reliability index under probabilistic loading β is 3.68, where it remains at this level until the onset of corrosion, and degrades to a final β of 2.05 over the 80 year period. For the same duration, an LRFD assessment would yield an initial safety index of 0.78 under *BS 153*, and have a final safety index of 1.36 under *Eurocode*; representing a load model producing a bending moment that is 36% over capacity of the section. If *BS 153* was used at year 80 for the LRFD assessment, the safety index would be 1.03; just 3% over estimated capacity and a 33 percentage point (pp) difference under the newer *Eurocode* model. Similar results can be seen for the RC beam and prestressed concreted bridges; where the beam bridge has a β of 4.79 at year 0 and 2.29 at year 80, and the prestressed concrete bridge has a β of 4.92 at year 0 and 3.51 at year 80. For the normative LRFD assessment, the RC beam bridge has an initial safety index of 0.68 under *BS 153* and 1.43 at year 80 under *Eurocode*. Again, the variation in the safety index at year 80 is significant, with a 45 pp difference to *BS 153* for a safety index of 0.98. For the prestressed bridge, the year 0 and 80 safety indices were 0.72 and 1.26, respectively; where the 1.26 was 38 pp difference over a year 80 safety index of 0.88 under *BS 153*.

While these end variations in safety index are somewhat large, it is perhaps an inappropriate comparison to compare the safety indices determined under *BS 153* and *Eurocode*, as these codes never operated sequentially. Thus, it is important to consider these 'jumps' in safety index as these occur in the transition from one code to the next. Two large jumps are seen over the 80 year period; when *BD 21/84* replaced *BS 5400* and when *Eurocode* replaced *BD 37/88*. When *BD 21/84* was introduced, the revised safety indices for the slab (1.01), beam (0.96), and prestressed (0.92) concrete bridges increased by 15 pp, 19 pp, and 17 pp over their *BS 5400* values, respectively. This change caused violation of the flexural limit-state for the slab bridge, and near violation for the beam and prestressed bridges. When the next jump occurs, at the introduction of *Eurocode*, these limit-states would have been violated due to the ongoing deterioration due to corrosion, if rehabilitation hadn't taken place.

Conversely, at the time of these 'jumps' (years 49 and 65), reliability assessment under the probabilistic load model gives values of β for the slab, beam, and prestressed bridge of 3.03, 3.83, and 4.49, respectively, at year 49, and 2.54, 3.05, and 4.02, respectively, at year 65. As maintenance and intervention decisions are often based on performance indicators such as β or the safety index, more commonly, the decision to intervene structurally on a bridge can be taken too hastily when code-defined loading is used instead of probabilistic loading, and lead to the misallocation of budgetary resources. Thus the use of β as the performance indicator over the lifetime of the bridge appears to provide a more stable assessment of safety than the normative LRFD assessment.

Furthermore, while the effects of deterioration on β have been shown over time (Figure 4.11), it is also worth examining the effect deterioration has on the importance factors α^2. Specifically, it is interesting to note the effect that this deterioration has on the ranking of the random variables in regard to α^2 (Figure 4.12). It can be seen that the variable X_{i1}, corresponding to the area of reinforcing or prestressing steel $A_{s,ps}$, experiences an increase in importance over time; this increase occurring after the onset of corrosion in the bridge. As $\sum \alpha^2 = 1$, any increase in α^2 for a specific variable is done so at the expense of other variables included in the model.

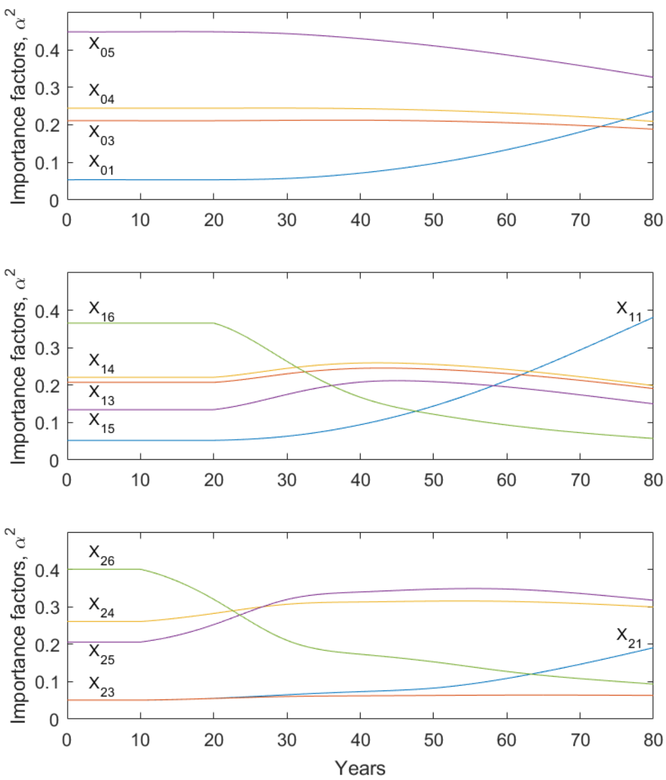

Figure 4.12: Variations in life-cycle importance factors for the a.) slab, b.) beam, and c.) prestressed concrete bridges

This is most prominent in the RC beam bridge, where X_{11} experiences its greatest increase of the three bridges, and is largely due to the decrease in α^2 of X_{16}; being the surfacing weight uncertainty factor λ_s. As per Chapter 3, this importance factor is quite high, due to the large CoV of 0.25 in the initial model. As the ongoing deterioration increases the uncertainty in X_{11}, it can be seen that X_{16} degrades to an importance measure that is more in line with its actual effect on the bridge, qualitatively. Interestingly, this same decrease in λ_s is seen in the prestressed bridge for the variable X_{16}, except in this case, X_{21} does not increase at a commensurate rate. This can be attributed to the prefabricated nature of prestressed beams, where there is less uncertainty in A_{ps} and while it is still subject to degradation, its uncertainty is not affected to the extent of it reinforced concrete counterparts. Instead, the concrete weight uncertainty factor $\lambda_c(X_{25})$ gains a measure of importance over time, which represents uncertainty in the permanent load of the bridge. As the bridge deteriorates and loses its flexural capacity, the effect of the self-weight of the bridge can become more of a consideration, thus explaining its increase in importance. It is noteworthy that the grade of steel used in the reinforcing and prestressing X_{i3} does not gain any significant importance over time.

4.5.4 Life-Cycle Cost Assessment

The life-cycle cost was evaluated in MATLAB (MathWorks 2015) using the model described in Section 4.4. The objective of the life-cycle cost model used here is to minimise C_{ET} while ensuring a minimum level of structural performance at all times, and then to determine the optimum inspection and repair strategy that achieves this goal. The structural performance is indicated through β and P_f, with a minimum performance indicator being the target reliability index β_T. In this assessment, β_T is set at a value of 2.5, which corresponds to a P_f of 0.0062. While this value is lower than those specified in Tables 2.2 and 2.3, it is being used as a nominal minimum value for convenience, from which it is possible to offer direct comparison between the various bridge capacities and their intervention requirements. Should a more onerous value be selected for β_T, such as 3.8, then only the bridge subjected to Eurocode loading would be above this threshold, and thus a comparison would not be able to be made for intervention requirements. As the expected cost of failure C_F is a function of the lifetime probability of failure $P_{f,life}$, and $P_{f,life}$ is heavily related to the number of inspections m conducted, it has been shown by (Frangopol et al. 1997) that C_F can be minimised by an optimum number of inspections m_{opt}. This is because only after an inspection can a repair activity be carried out; an activity which will improve β and lower P_f. Thus, as m increases, the likelihood of failure and thus $P_{f,life}$ and C_F are reduced. However, there exists a point of diminished returns when $P_{f,life}$ is low enough to keep C_F as a minor component of C_{ET}, and for increasing values of m, C_{ET} rises with the expected cost of inspection C_{INS}.

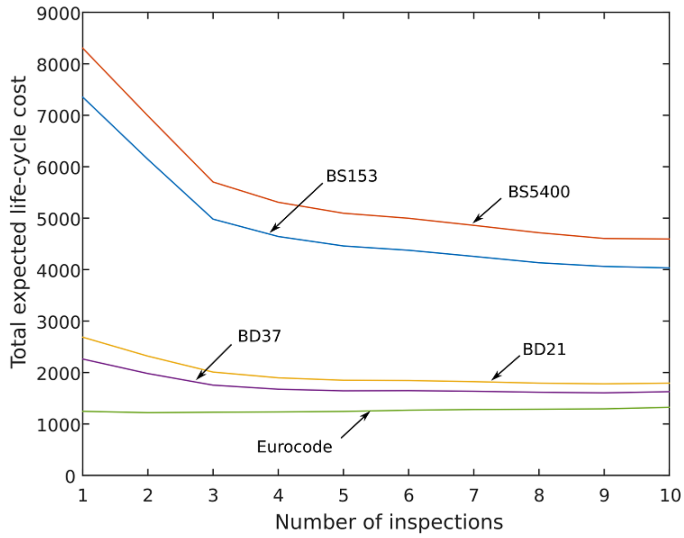

Figure 4.13: Variation in total expected life-cycle cost against increasing number of inspections for each design load

The effect of more onerous traffic load requirements on C_{ET}, as well as the effect of increased inspections m is shown in Figure 4.13. In the figure, there exists a gulf between C_{ET} for the earlier codes, *BS 153* and *BS 5400*, and that for the more modern codes; *BD 21*, *BD 37*, and *Eurocode*. Despite the required increase in C_T for the modern codes, the values of C_{ET} are significantly lower, and less dependent on the number of inspections carried out. This is explained by the effective reduction in C_F due to the improved β provided by the increased A_s demanded by the modern codes. The effect that this increased A_s has on C_{ET} can be seen in Figure 4.14. It is clear that while there is very little relative change in C_T, there is a significant reduction in C_{ET} to the point where the number of inspections m loses significant importance.

The effect of increasing values of m and A_s on C_{ET} and C_F can be visualised in the 3D surface plots in Figures 4.15 & 4.16, respectively. It can be seen in Figure 4.16 that C_F, and consequently $P_{f,life}$, lower to a near zero point for 10 inspections under *Eurocode* designed A_s. It is clear from these figures that the increased demand of A_s, and thus the small relative increase in C_T, has a greater influence on C_{ET} than the inspection regime.

This contention can be borne out by evaluating the repair strategies for each bridge, based on the number on inspections that return a minimum value of C_{ET}. From Figures 4.13–4.15, this can be seen to result in m_{opt} of 9, 9, 5, 5, & 2 for *BS 153–Eurocode*, respectively. In assessing the repair options available, two strategies were adopted: to repair the structure to its original state after each

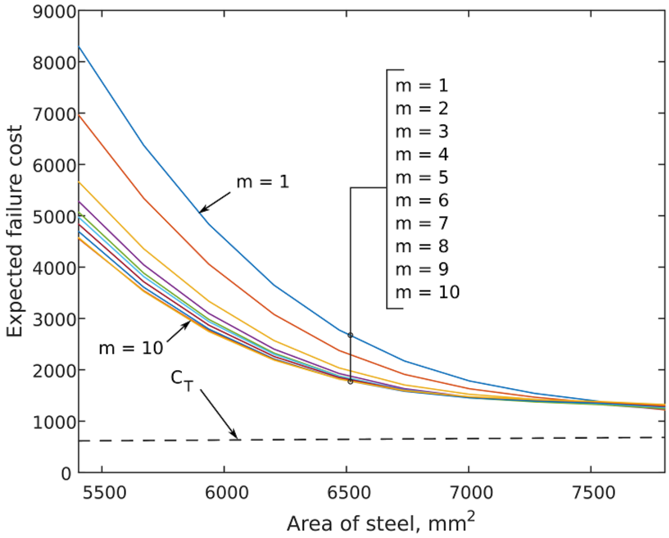

Figure 4.14: Diminishing total expected failure cost for increasing area of steel and increasing number of inspections

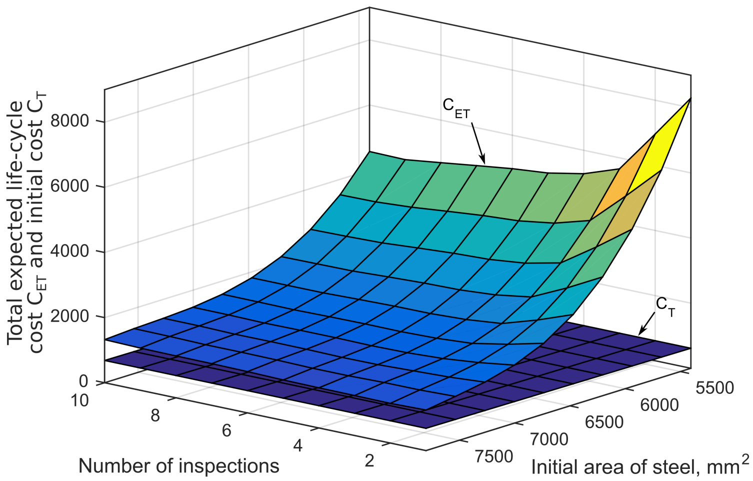

Figure 4.15: Effects of increasing inspections and area of steel on total expected life-cycle cost

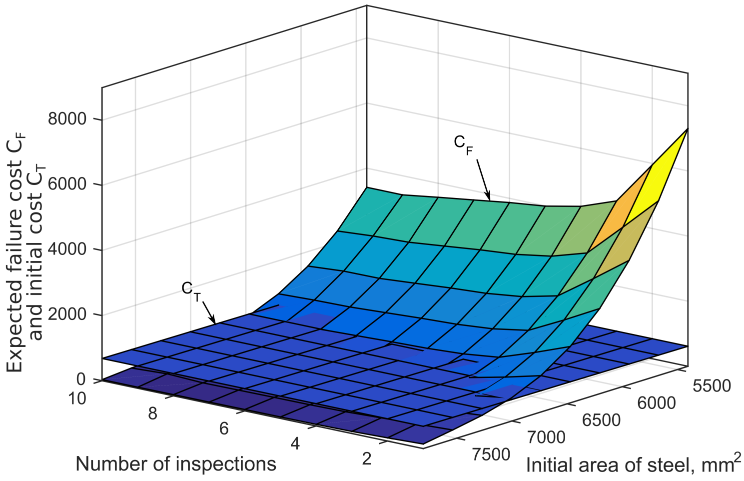

Figure 4.16: Effects of increasing inspections and area of steel on total expected failure cost

inspection, and to repair the structure to its original condition at a time where it would fall below β_T before the next scheduled inspection. For both these strategies, a uniform inspection interval was assumed. In this model, it was assumed that once chloride induced corrosion began in the original structure, it would continue to act on A_s immediately after repair.

The result of the first strategy can be seen in Figure 4.17. Here the bridge is repaired prematurely for those designed under modern standards, resulting in a misallocation of resources, but is seemingly appropriate for the bridges designed by the less onerous standards. It is evident from Figure 4.18 that the second repair strategy results in a more sensible schedule of repair activities, to the point where the *Eurocode* designed bridge will maintain a level above β_T for the 80 year assessment period, under the presented deterioration model. Furthermore, bridges designed under *BD 21* and *BD 37* require only one repair activity, whereas those designed under *BS 183* and *BS 5400* require 8 and 9, respectively. This is further evidence of apparent life-cycle cost savings available with a small increase in the initial investment in the structure. As the design codes under which a bridge is constructed have an apparent effect on the levels of maintenance and intervention required later in the bridges life-cycle, bridges designed during periods of transition between one design code to another require critical evaluation; as while their relative age would be similar, there could be significant, code-based discrepancies between their strengths, even when constructed in close proximity of years.

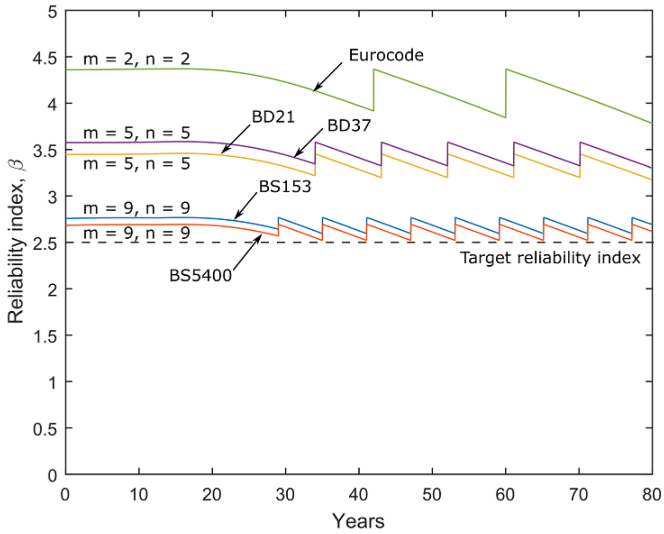

Figure 4.17: Effect of non-optimum repair strategy on β for uniform interval inspection for each design load

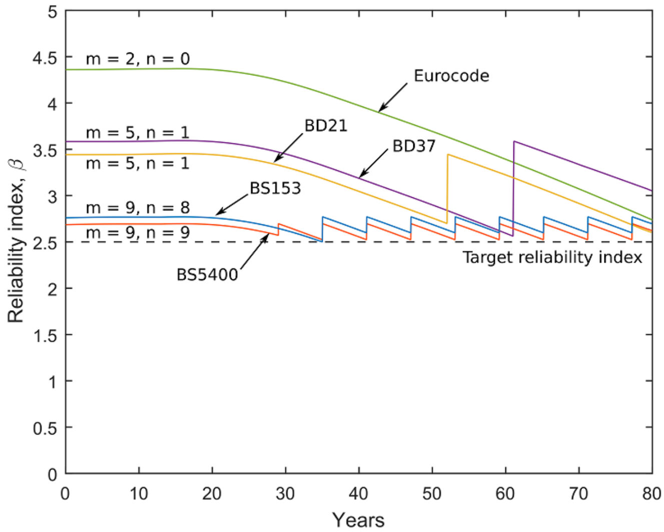

Figure 4.18: Effect of optimum repair strategy on β for uniform interval inspection for each design load

4.6 Conclusions

This chapter addresses research Objectives 2 and 3 in its investigation of the life-cycle performance and costs effects of different codes of practice for bridge design and assessment. An equivalent structural reliability analysis, based on information available only from the code definitions and their variations, was conducted on three bridges to assess the effect of changing definitions of code-defined traffic loading on safety classifications of the structures. These results were compared with those for site-specific probabilistic loading to determine how representative the safety classification for a bridge assessed under specified loading was against a more realistic loading scenario. It was observed that earlier codes produced less onerous flexural load effects and, as such, resulted in a reduced demand for flexural capacity and, thus, reliability indices closer to that determined under the probabilistic load model. This, however, results in a situation where bridges designed and assessed under these early codes are regularly being subjected to close to their ultimate loads. As these code-defined loads were said to have a large return period, such proximity between the 'typical' and 'ultimate' loading is not an expected or desirable scenario.

It was shown that bridges produced under loading prescribed by modern standards produced bridges with a higher β assessed under a probabilistic load model, and resulted in a significantly reduced expected life-cycle cost, despite the increased initial construction costs due to a higher minimum requirement for flexural reinforcement. This increased initial cost was seen to be significantly offset with a lower expected cost of failure, which is a function of the probability of failure and thus the reliability index β. This gives rise to the question as to whether there is an optimum point at which the initial cost can be increased to minimise the total expected life-cycle cost, and is there further variables that can be optimised at the design stage for bridges. Furthermore, the practical ways such a philosophy can be adopted in normative standards are unanswered; be it through refinements of partial factors for resistance variables such as the area of steel A_s or the compressive strength of concrete f_c, or through the a more holistic increase in safety factors regarding the applied traffic loading. Despite these remaining issues, it is clear from the presented results that there is scope for significant savings through a more conservative approach at the design stage.

Given the disparity between β for the probabilistic load model and the more recent codes of practice, it is evident that bridge structures designed and constructed according to these standards should have a higher resistance capacity than seen in bridges designed to the extent of the earlier standards. It can thus be suggested that bridges designed to the extent of the modern standards will perform better in terms of β when assessed against a probabilistic load. However, since many of the assessments tend to use code based definitions of design traffic loading, their direct use in assessment of existing bridges is not best practice for economical life-cycle asset management. The use of probabilistic load modelling, such as through site-specific weigh-in-motion (WIM), in reliability analyses yields a more accurate assessment of the true safety of a bridge.

Chapter 5

Multivariate Data Techniques for Network Condition Monitoring

5.1 Introduction

5.1.1 Overview

The previous chapters have dealt with the theory and application of the structural reliability method and its role in the assessment of existing bridges. It was seen how sensitive the method can be to varying levels of information in the input model, and the pitfalls that can compromise a useful and sophisticated method in its application for life-cycle assessment. To reiterate a core concept in this thesis, structural reliability methods can be said to refer more to our state of knowledge of the structure, which is directly dependent on uncertainty and the level of information available for the model. In this regard, efforts to better inform our probabilistic models rely heavily on the gathering of data and information to reduce uncertainty in the limit state equations. While methods to reduce the uncertainty in the loading model S are largely concerned with bespoke, site-specific probabilistic loading models, various methods exist from which to try to reduce the uncertainty in R. Many of these methods involve non-destructive evaluation (NDE) of the bridges elements, or through Bayesian updating of β based on expected values of condition rating against those observed in practice. In modern bridge management systems (BMS), the most plentiful source of information comes in the form of condition rating data evaluated through visual inspection. However, while this typically represents the largest amount of data available for bridge management, it gives no information on the structural capacity of these bridges. That being said, there are opportunities to use this data to a greater extent to extract the greatest knowledge of the state of a network of bridges. Methods to exploit the vast amount of data available to bridge managers on the state of existing bridges have not been explored to date. In this chapter, for the first time, large volumes of condition rating data will be explored in an effort to extract patterns within these datasets from which to improve our knowledge on the state of the resource variable R in the basic reliability model.

5.1.2 Background

Infrastructure asset management involves management of the expectation of infrastructure stakeholders at different levels (Weninger-Vycudil et al. 2015, Lloyd 2010). In the past, a strategy of 'deferred maintenance' (Petroski 1996) on national bridge stock has resulted in a scenario where, in the United States alone, 11% of the nation's bridges are said to be structurally deficient, while 25% are described as being 'functionally obsolete' (ASCE 2013). As such, the maintenance of an infrastructure network is a resource intensive endeavour which seeks to mitigate the risk of its failure to the society and economy (Mueller and Stewart 2011, Denysiuk et al. 2016). For a country's national bridge stock, the management of these maintenance activities are often enabled using bridge management systems (BMS), which provide a platform to coalesce various assessment methods and criteria in order to improve intervention planning by asset managers (Hearn 1998, Lauridsen et al. 1998, Matos et al. 2005). This allows an organisation to combat the so-called "asset time bomb" (Thurlby 2013).

The most prevalent methods of assessment for these systems is through scheduled visual inspection activities, from which the damage state of the asset can be described through condition ratings; the results of which often guide further assessment and, eventually, intervention, (Das 1998, Estes and Frangopol 2003). The populating of these systems also leads to the storage of large quantities of so-called metadata; which can describe the physical parameters of the bridge.

When applied to a national bridge stock, this results in a large database of condition ratings and provides scope for which to apply modern principals of "big data analysis" (Manyika et al. 2011, Kobayashi and Kaito 2016). When examining large amounts of data in this way, it is possible to use advanced multivariate analysis methods to extract patterns within the data-set and to define underlying latent constructs for variables within this data-set using data-reduction techniques. For a BMS, these variables are in the form of condition ratings for a bridges individual elements. The application of multivariate methods such as principal component analysis (PCA) to bridge management systems has been shown to be successful in describing a large asset base (Hanley et al. 2015), as well as differentiating between condition state signatures between different bridge types (Hanley et al. 2016b). In this chapter, a background to a representative BMS is presented, including an explanation of the condition rating descriptions used by Ireland and Portugal. The application of multivariate data-reduction to compare two networks from different regions is evaluated on data-sets containing condition rating data for a large number of masonry arch bridges in Ireland and Portugal; which are typical for both countries. The appropriate data size required for a reasonable comparison between two data-sets is established, and the methodology for conducting the multivariate analysis is presented.

5.2 Bridge Management Systems

5.2.1 Background

Due to the societal importance of bridges, early infrastructural asset management systems have primarily been based around bridge management, and thus many systems concerned with other infrastructural objects follow from the development of BMSs (Mirzaei et al. 2014). A BMS is a rational, popular, and systematic approach to carrying out all management activities related to managing a network of bridges (Scherer and Glagola 1994), including the prioritisation of bridges for intervention activities. A complete BMS defines a set of interrelated codes and guidelines for bridge management activities, and an organisation structure to plan and implement these activities; as well as a computational tool to track, record, and process the results of these management activities (Lauridsen et al. 1998).

While the maintenance requirements of any bridge, or a network thereof, are best evaluated in relation to its ability to perform its structural and functional role, too often the underlying assumptions of the use of a BMS are that a bridges maintenance needs are dictated by its condition state, and that the most appropriate maintenance actions are those which will cost more at a later date, when deterioration is allowed to continue (Das 1998). A typical, modern BMS comprises a number of basic components, namely: inventory, inspection, maintenance, financial, and condition rating; all comprised in a central database (Ryall 2009). The inventory data recorded in the BMS details the bridges location, construction type, crossing type, etc., and inspection data comprises visual inspection condition ratings for various defined elements within the overall bridge structure, recorded by trained bridge inspectors. The condition ratings of the elements are indicative of any damage present in the element and are used to assign an overall condition rating to the bridge; a rating that has been shown to be sensitive to the worst condition rating of the primary structural elements in BMSs in which the overall condition rating is not a linear combination of the elemental condition ratings (Hanley et al. 2016b).

5.2.2 Condition Evaluation

The prevalence of condition ratings in BMSs has a historic reason in that when these modern systems were developed, the most plentiful type of electronic information available for bridge damage states was in the form of discrete, numerical condition ratings, and thus modern systems needed to be designed around existing platforms and protocols (Hearn 1998).

There are various methods used in BMSs for determining a bridges overall condition rating from visual inspection data; with two common methods being the worst-conditioned element approach and the weighted averaging approach (Swanlund 2016). Both methods are fundamentally based on assigning numerical condition ratings to individual bridge elements during visual inspection, based on a well-defined scale of damage present (Table 5.1).

The worst-conditioned element approach is based on the principle whereby the overall condition rating of the bridge is determined as being equal to the worst condition rating of the primary structural elements of the bridge; thus, significantly tying the overall condition rating to that of a single element, and not to the total amount of damage present in the bridge. While this is useful for identifying bridges that are vulnerable to elemental failure, it results in a scenario where similar bridges of vastly different states of distress can possess equal

Table 5.1: Condition rating descriptions, adapted from NRA (2008)

Rating	Description
0	No or insignificant damage.
1	Minor damage but no need of repair.
2	Some damage, repair needed when convenient. Component is still functioning as originally designed. Observe the condition development.
3	Significant damage, repair needed very soon. i.e. within next financial year
4	Damage is critical and it is necessary to execute repair works at once, or to carry out a detailed inspection to determine whether any rehabilitation works are required.
5	Ultimate damage. The component has failed or is in danger of total failure, possibly affecting the safety of traffic. It is necessary to implement emergency temporary repair work immediately or rehabilitation work without delay after the introduction of load limitation measures.

condition ratings. For management of maintenance activities, the overall state of deterioration of the bridge is required for adequate planning and resource allocation. When a small number of these bridges are compared against each other, it is simple and trivial to discern which is in the worst state. However, in BMSs that contain a large number of bridges, such a comparison becomes cumbersome and unwieldy.

To overcome this, the weighted averaging approach assigns an element importance factor to the condition ratings of each element; often based on engineering judgement alone. The overall condition rating is then determined as a function of the elemental condition ratings and weighting factors; these functions often differing from region to region, based on the requirements of asset managers and stakeholders. This method has the benefit of establishing an overall damage assessment of the bridge, and is not as sensitive to the damaged state of one element in the way the worst-conditioned element approach is. Due to this, it can be considered a more powerful approach in the planning of future maintenance activities, as the most critical bridges are identified in the BMS, as opposed to those bridges with the most critical elements. However, the use of engineering judgement alone in the definition of the weighting factors itself leads to issues of bias and inaccurate assessments of overall damage state. In the following chapter, multivariate analysis techniques will be shown to use existing elemental condition rating data on a large scale to help define, as a first step, information-based, structure-specific weighting factors.

5.3 Multivariate Data-Reduction Techniques

5.3.1 Background

The purpose of data-reduction techniques in multivariate analysis is to find a suitable lower-dimensional space with which to represent the original data, allowing for the discovery of data structures and patterns, and to enable visualisation of the data in two- and three-dimensional spaces (Martinez et al. 2011). Data-reduction techniques create new variables y_i that are functions of the original variables x_i in the data-set, and thus establish the relationship between these original variables $(x_i, x_{i+1}, \ldots, x_n)$. Common methods for data-reduction are *principal component analysis* (PCA) and *exploratory factor analysis* (EFA). While both methods typically yield similar results, they have differing levels of computational effort and efficiency. A brief overview of each method follows for comparative purposes.

5.3.2 Principal Component Analysis

PCA is a multivariate, data-reduction technique, the primary purpose of which is to reduce the dimensionality of a data-set and to redefine the input variables as PCs, or latent variables, being a linear combination of the original variables (Mardia et al. 1979, Jolliffe 2002). In defining these PCs or new variables, the goal is to have a magnitude less than the variables in the original data-set, but while preserving most of the information contained within it. This is accomplished by highlighting the variables that demonstrate the most variance in the data set. The first principal component Y_1 is defined as:

$$Y_1 = \boldsymbol{\alpha}_1'\mathbf{x} = \alpha_{11}x_1 + \alpha_{12}x_2 + \cdots + \alpha_{1p}x_p = \sum_{j=1}^{p} \alpha_{1j}x_j \tag{5.1}$$

Where $\boldsymbol{\alpha}_1'\mathrm{x}$ is a linear function of the elements $\mathbf{x}$ having maximum variance, and $\boldsymbol{\alpha}$ is a vector of p coefficients α. The sum of the square of the coefficients α_i is equal to unity, and is a better indicator of the influence the coefficient has than the raw value:

$$\sum_{i=1}^{p} \alpha_i^2 = \alpha'\alpha = 1 \tag{5.2}$$

The first PC Y_1 is the direction along which the data set shows the largest variation (Ringnér 2008), and the second PC Y_2 is determined under the constraint of being orthogonal to Y_1 and to have the largest variance (Abdi and Williams 2010). The second PC $Y_2 = \boldsymbol{\alpha}_2'\mathbf{x}$ is found in a similar manner to Y_1, and so on for the subsequent PCs, up to *p* PCs, Y_p. It is desired, however, that most of the variance in the data set is accounted for in the PCs $\ll p$, in that dimensionality reduction is the primary aim of this method. In order to locate the PCs, it is necessary to determine the covariance matrix $\boldsymbol{\Sigma}$ of the vector of random variables $\mathbf{x}$. It can then be shown that $\boldsymbol{\alpha}_k$ is an eigenvector of $\boldsymbol{\Sigma}$ corresponding to its *k*th largest eigenvalue λ_k (Jolliffe 2002).

The above can be discussed in matrix terms where a PCA can be conducted through an eigenvalue decomposition (EVD) or a more robust and generalized singular value decomposition (SVD) (Chambers 1977). For a data matrix **X** of *n* observations on *p* variables measured about their means:

$$\mathbf{X} = \mathbf{ULA}' \tag{5.3}$$

Where **L** is an $(r \times r)$ diagonal matrix, and **U** and **A** are $(n \times r)$ and $(p \times r)$ matrices, respectively, with orthonormal columns, and *r* is the rank of **X**. It has been observed that SVD approach to PCA is a computationally efficient and generalised method to determining the PCs. Further discussion of the method can be seen in depth with Gower (1966), Anderson (1963), Wold et al. (1987), Hui Zou et al. (2006), Rao (1964), Tipping and Bishop (1999).

5.3.3 Exploratory Factor Analysis

Factor analysis is a multivariate technique for which the primary purpose is to define the underlying structure amongst variables in a data-set, and to establish factors; which are sets of highly correlated variables. These factors cannot be observed directly.

$$x_i = \sum_{i=1}^{k} \lambda_{ij} f_i + u_i + \mu_i, \quad i = 1, \cdots, p \tag{5.4}$$

Where f_i is an underlying common factor, λ_{ij} are factor loadings, u_i are random disturbance terms, and μ_i is a mean value of x_i (Mardia et al. 1979). This equation can be written in matrix form as:

$$\mathbf{x} = \mathbf{\Lambda f} + \mathbf{u} + \mathbf{M} \tag{5.5}$$

Where $\mathbf{x}$ is a $(p \times 1)$ random vector with mean $\mathbf{M}$ and covariance matrix $\mathbf{\Sigma}$, $\mathbf{\Lambda}$ is a $(p \times k)$ matrix of constants, and $\mathbf{f}$ and $\mathbf{u}$ are $(k \times 1)$ and $(p \times 1)$ random vectors, respectively. This model is valid for $\mathbf{x}$ when $\mathbf{\Sigma}$ can be decomposed in the form of:

$$\mathbf{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi} \tag{5.6}$$

Where $\mathbf{\Psi} = \mathbf{diag}(\psi_{11}, \cdots, \psi_{kk})$ is the covariance matrix of $\mathbf{u}$. The variance σ_{ii} of $\mathbf{x}$ can be defined as:

$$\sigma_{ii} = h_i^2 + \psi_{ii}, \quad h_i^2 = \sum_{j=1}^{k} \lambda_{ij}^2 \tag{5.7}$$

Where h_i^2 is the variance of x_i shared with the other variables via common factors f_i, and ψ_{ii} is the variance of x_i not shared with the other variables due to the unique factor u_i. These are referred to as communality and specific variance for h_i^2 and ψ_{ii}, respectively.

In order to give the factors $\mathbf{f}$ intuitive meaning, it is necessary to rotate them to a more interpretive space. A popular method is the varimax rotation; which is an orthogonal rotation in an iterative manner. The matrix of rotated loadings $\mathbf{\Delta}$ is given by:

$$\mathbf{\Delta} = \mathbf{\Lambda G} \tag{5.8}$$

Where $\mathbf{G}$ is a $(k \times k)$ orthogonal matrix that maximises the function ϕ, which is the sum of the variances of the squared loadings within each column of the loading matrix.

The calculation of factor scores f can be conducted using two popular methods; Bartlett's method for deterministic scores $\hat{f}$, and Thompson's method for stochastic scores f^*. Using Bartlett's method, the factor score can be determined as:

$$\hat{\mathbf{f}} = (\mathbf{\Lambda}'\mathbf{\Psi}^{-1}\mathbf{\Lambda})^{-1}\mathbf{\Lambda}'\mathbf{\Psi}^{-1}\mathbf{x} \tag{5.9}$$

The use of deterministic factors is appropriate for use on a specifically defined data set, whereas stochastic Thompson factors are favoured when sampling from a population of data.

5.3.4 Comparison of PCA and EFA

A detailed comparison of the two methods was conducted by Velicer and Jackson (1990), who generalised both methods into the equation:

$$\eta = \mathbf{A}\zeta + \varepsilon \tag{5.10}$$

Where η is a set of p random variables, ζ is a set of m random variables, of which $m \leq p$, ε is a set of p residuals, and $\mathbf{A}$ is the $p \times m$ multiple regression pattern for optimally predicting the variates η from the m variates in ζ. In PCA, the covariance matrix of ε cannot be diagonal and of full rank, whereas in EFA, this covariance matrix must be diagonal and of full rank.

There are many similarities between the methods and results for PCA and EFA, in that they both attempt to describe a data-set with a reduced number of variables than in the original set, with comparable results between the methods for highly correlated data. However, there does exist differences between the methods, with a primary difference being that retained principal components (PC) account for maximal variance, whereas factors account for common variance. In a comparison of the two methods, Suhr (2005) recommended the most appropriate application of these techniques involved selecting the preferred method *a priori*, and not to conduct both PCA and EFA on the same data-set. Velicer and Jackson (1990) argued that while well modelled data-sets are likely to produce similar results when either method is applied, PCA has advantages over EFA in that it is more robust in terms of determining the number of components to retain, as well as its computational efficiency when applied to large data-sets.

The methods differ in that PCA has a unique solution, and the composition of a PC is not sensitive to the number of components retained; whereas in EFA, there is no unique solution in that the composition of the factors are variant with the number of factors specified and retained. The rules regarding the retention of components and factors are similar for the two methods; with popular rules being the scree procedure (Cattell 1966), eigenvalue greater than unity procedure (Kaiser 1960), and the minimum average partial correlation procedure (Velicer 1976). While the Kaiser rule is the simplest to implement, it can lead to problems of retaining too many components/factors, or overextraction (Browne 1968). While, in practical purposes, this is less of a problem for PCA which has a unique solution, it can cause problems for EFA whereby the number of factors retained, selected *a priori*, directly affects the results of the analysis. In fact, differences in the results between the two methods can often be attributed to the number of factors retained in EFA. Thus, PCA is the preferred method of use for this analysis.

5.4 Methodology

5.4.1 Asset Base

In order to test the feasibility of applying multivariate data reduction to condition rating data from a BMS, data was collected from two sources: a data-set of 3,036 bridges from the Portuguese national bridge authority, *Infraestruturas de Portugal* (INFRAPOR), and a data-set of 458 bridges from the regional authority responsible for non-national bridges in Cork, *Cork County Council* (CCC). This allows a comparison of data-sets at a national and regional level in Europe, both of which operate under similar BMS structures. Within these data-sets, the entirety of the bridges from the CCC data-set were masonry arch bridges, whereas the data from INFRAPOR contained a greater variety of bridge construction; with the predominant types being reinforced concrete (1690) and masonry arch (713) bridges. For each bridge in both data-sets, the condition state was evaluated for each element on the 0-5 scale described in Table 5.1, and the overall condition of the bridge was determined using the worst-conditioned element approach described in this chapter.

Within these data-subsets, the problem of 'missing data' needed to be accounted for (Little and Rubin 2002). In multivariate analysis, it is often possible to use the existing structure of the data to estimate the missing data and complete a data-set. This can simply be accomplished through element-wise computation of the correlation matrix (Beale and Little 1975) or through replacing empty values via imputation; usually selected as the mean value for the variable, or by a sampling procedure where the replacement value is selected m times from a probability distribution and a PCA is conducted for each m (Schafer 1997). Further advanced methods to solve this problem involve use of the maximum likelihood estimation (MLE) method through the expectation-maximisation (EM) algorithm (Little and Rubin 2002, Dempster et al. 1977). However, in the case of BMSs, an absent condition rating for a specified element is typically due to this element not being present on the bridge, and therefore it is neither appropriate nor accurate to use the above methods to complete the data-set.

The account for this, the PCA was conducted on the specified bridges, but was limited only to the primary elements that are to be expected on these bridges. The primary elements can be said to be structural and non-structural. Structural elements are typically the deck, abutments, walls, and piers. Primary non-structural elements are usually parapets/guardrails, embankments, surface, and bearings. For the single-span masonry arch bridges, there was no necessity for condition rating data for piers and bearings, and so the analysis was conducted for: abutments, parapets, walls, surface, deck, and embankments. The reinforced concrete bridges were also single-span, but as there was little information available on the condition of the bearings, the PCA was conducted for the same variables across both regions and both bridge types. The exclusion of the other elements does not compromise the PCA, as a PCA does not look for hidden correlation in the variables, but for highest variation in the presented variables.

5.4.2 Required Sample Size

To compare two data-sets of differing sizes, it is necessary to determine the magnitude of data-set at which point the analysis stabilises and converges to a reliable solution. Being the larger data-set of the two presented, the Portuguese data was used for this bootstrapping. A Monte Carlo Simulation was used to randomly select 100 bridges, 50,000 times, from the total population size, and a PCA was conducted for each of these 50,000 samples. This was repeated for 200, 300, up to 500 bridges, for a total of 250,000 individual PCAs. From this, it was possible to establish a histogram of PC coefficients α_i for each variable, for each sample size m. It was observed that these histograms could be reasonably well-fitted to a normal distribution (Figure 5.1).

For this test, it can be seen that the mean values and standard deviations $\mu(\sigma)$ of α_{11} are 0.4613(0.0632), 0.4633(0.0393), 0.4645(0.0275), 0.4650(0.0188), and 0.4654(0.0112) for $m = 100 \to 500$, respectively (Figure 5.2). The narrowing of the probability density function (PDF) and the resulting reduction in coefficient of variation (CoV) show, as expected, that the μ stabilises with increasing m. However, there is a diminishing returns observation in that the rate of convergence to the stable value of α_{11} slows significantly after $m = 300$, and

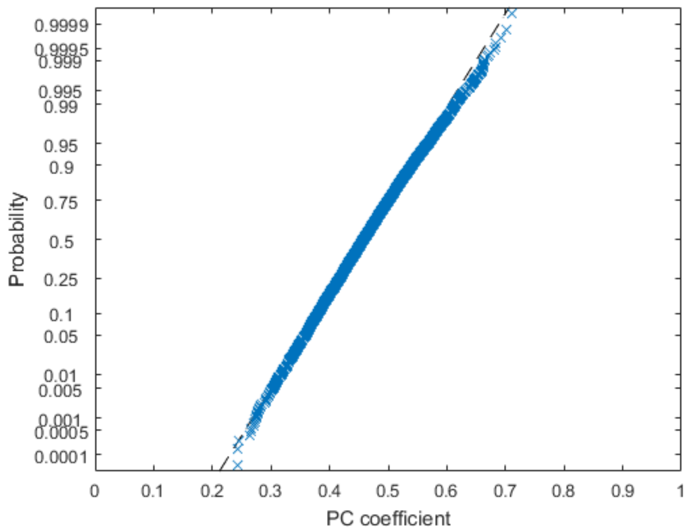

Figure 5.1: Normal probability plot of α_{11}.

this can be seen when compared to α_{11} for the total data-set in terms of relative error (Figure 5.3).

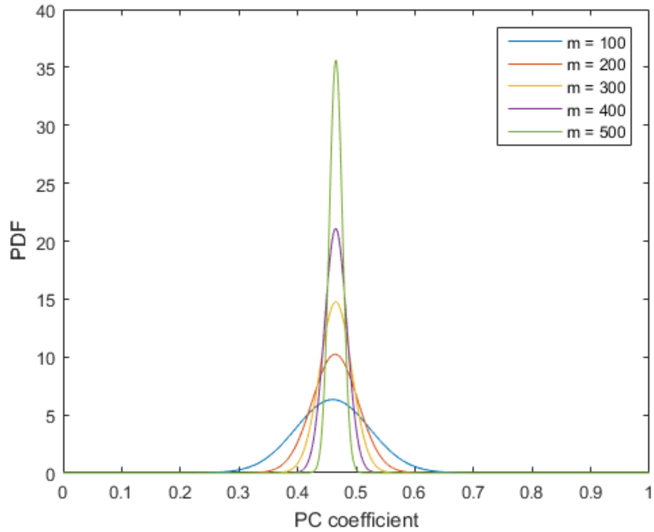

Figure 5.2: Probability density functions of α_{11} for increasing number of bridges m.

While this convergence is presented for a single coefficient α_{11} in the first PC Y_1, the similar stabilisation and convergence rates were observed for the remaining results of α_{ip}. As this stabilisation point for m is observed to be less than the size of the individual data-sets, it is thus possible to compare the results of a PCA for both regions. The quartile distribution of α_{ip} each element for $m = 300$ can be represented in a boxplot, where the box represents the 25th to 75th percentile

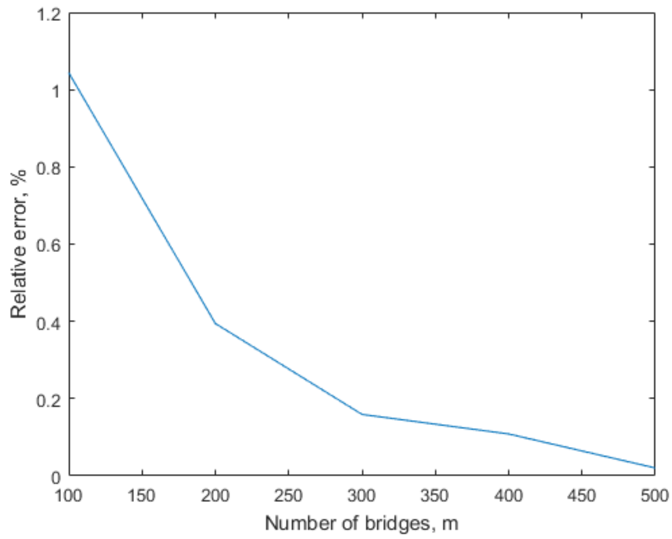

Figure 5.3: Relative error of α_{1p} for increasing number of bridges m.

range, encompassing the median, with the extents to the most extreme data points not considered outliers (Figure 5.4).

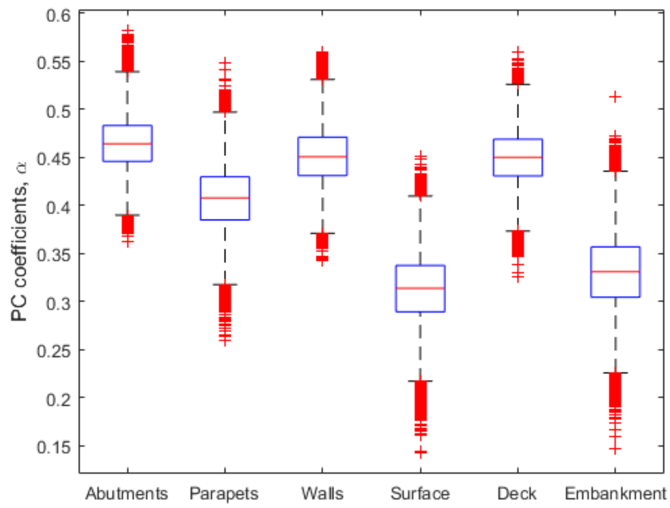

Figure 5.4: Quartile distribution of α_{1p} for $m = 300$.

5.5 Interpretation of PCA

5.5.1 Dimensionality Reduction

As the traditional purpose of PCA is dimensionality reduction of a data-set, it is important to consider the criteria for which an appropriate number of PCs can be said to represent the original data-set. For the most part, the most extensively used procedures or rules for determining the appropriate number of PCs are informal, and largely based on judgement. The most subjective of these procedures is to use a scree plot of the eigenvalues (Cattell 1966) of the data in order to visualise the number of important PCs. In this method, as the slope of the scree plot begins to flatten, the PCs become less important as they retain less variance then the previous PCs, and thus can be discarded from the data-set. The point on the plot where there is a marked difference in slope on either side of a PC is said to be the elbow of the plot, and the corresponding PC is the last PC to be retained in the analysis. An example of this plot is shown in Figure 5.5 and detailed in Jolliffe (2002), where it can be seen that the elbow of the plot occurs at the 4th PC, with the plot flattening up to the 7th PC. Using this approach would suggest that the first four PCs are appropriate to retain in the analysis.

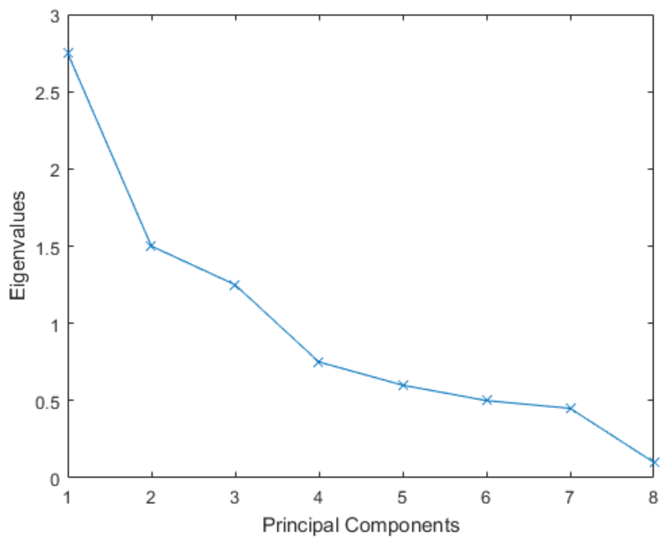

Figure 5.5: Scree plot example, reproduced from Jolliffe (2002)

Another method of determining how many PCs to retain is named the *Kaiser Rule*, and it simply states that PCs that have eigenvalues λ less than 1 should be discarded from the analysis (Kaiser 1960). This rule is based on the analysis of independent variables in **x**, whereby the correlation matrix will contain unit variances, and that any PC with a value of λ less than 1 will contain less information than the original variables. It has been suggested by Jolliffe (1972), however, that choosing a threshold less than 1 would be more optimal as it would eliminate errors associated with using samples of data. This might arise when a single variable is independent of every other variable in **x**, and thus produces a PC with a λ of close to 1 that contains information independent of the other PCs. When sampling this variable from a larger population, it is likely that this λ may be less than 1 due to sampling error, and strict adherence to the *Kaiser Rule* would discard useful information. In that study, Jolliffe (1972) suggested that a threshold for λ of 0.7 would be more appropriate than 1.

The last method to discuss here is to select the number of PCs to be retained based on the total, cumulative variation present in a minimum number of PCs. In this method, it is desired to retain PCs that contain between 70–90% of variance of **x**; working from largest variance to smallest. This rule is based more on judgement than any strict adherence to a minimum value or threshold; and the above recommended boundary of variance should be employed based on judgement and requirements of the analysis based on the original data **x**.

It is possible to combine the above rules in a single Pareto chart, which allows concise visualisation of all three criteria: scree slope, eigenvalues, and total cumulative variation. The data in Figure 5.5 can be reconstituted into a Pareto chart as an example (Figure 5.6). It can be seen in this figure that while the first four PCs account for 79% of the variance, including a further PC or two would include 87% and 93% of the variance, respectively. However, retaining anything beyond three PCs would violate the *Kaiser Rule*, while anything beyond four PCs would not comply with Jolliffe's amended threshold of 0.7.

This shows the relative subjectivity of determining the extent to reduce the dimensionality of a data-set. However it does not pose an obstacle to analysis as the number of retained PCs does not affect the results of a PCA; whereas, analogously, the same cannot be said for factor analysis, in which the results of the process is dependent on the number of factors to be retained, which are selected *a priori*. In the results to follow in the next chapter, the Pareto chart will be used as a basis for selecting the number of retained components; keeping in mind the above rules and best practice.

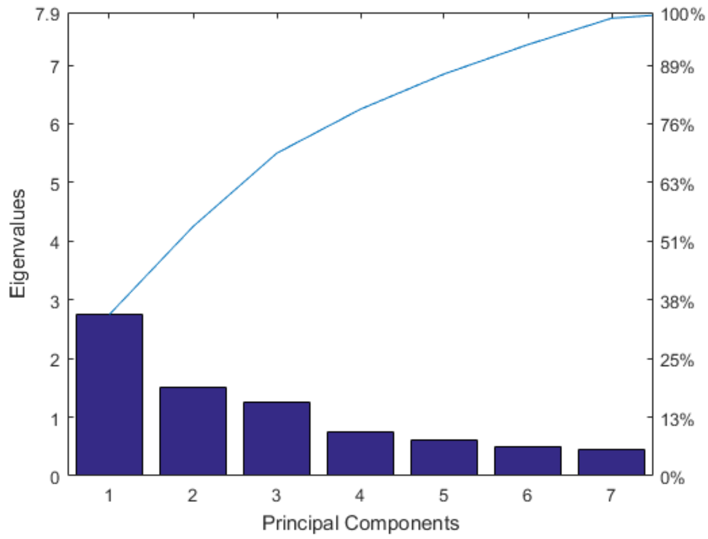

Figure 5.6: Pareto chart of data from Figure 5.5

5.5.2 Principal Component Coefficients

In the previous section, where it was shown how the eigenvalues λ_j of the data-set represent the level of importance attributed to the corresponding PCs, in this section it will be briefly shown how the associated eigenvectors α_j contribute to the new latent variables Y_j. In this thesis, these eigenvectors will be referred to a PC coefficients.

Referring back to Equation 5.1, a linear function $\boldsymbol{\alpha}_1'\mathbf{x}$ of the elements of $\mathbf{x}$ having maximum variance is sought; such that the vector $\boldsymbol{\alpha}$ maximises:

$$\mathrm{var}[\boldsymbol{\alpha}_1'\mathbf{x}] = \boldsymbol{\alpha}_1'\boldsymbol{\Sigma}\boldsymbol{\alpha}_1 \tag{5.11}$$

Under the constraint of $\boldsymbol{\alpha}_1'\boldsymbol{\alpha}_1 = 1$. The standard approach for this maximisation (Jolliffe 2002) is to use the technique of Lagrange multipliers, whereby:

$$\boldsymbol{\alpha}_1'\boldsymbol{\Sigma}\boldsymbol{\alpha}_1 - \lambda\left(\boldsymbol{\alpha}_1'\boldsymbol{\alpha}_1 - 1\right) \tag{5.12}$$

is to be maximised, with λ being a Lagrange multiplier. Differentiating with respect to $\boldsymbol{\alpha}_1$ gives:

$$\begin{aligned} \boldsymbol{\Sigma}\boldsymbol{\alpha}_1 - \lambda\boldsymbol{\alpha}_1 &= 0 \\ \left(\boldsymbol{\Sigma} - \lambda\mathbf{I}_p\right)\boldsymbol{\alpha}_1 &= 0 \end{aligned} \tag{5.13}$$

Which is the general form of the eigenvalue equation, with $\mathbf{I}_p$ being the $(p \times p)$

identity matrix. This shows that λ is the eigenvalue of $\mathbf{\Sigma}$, with $\boldsymbol{\alpha}_1$ being the corresponding eigenvector. Referring back to the maximisation Expression 5.11, it can be rewritten as:

$$\boldsymbol{\alpha}_1'\mathbf{\Sigma}\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_1'\lambda\boldsymbol{\alpha}_1 = \lambda\boldsymbol{\alpha}_1'\boldsymbol{\alpha}_1 \tag{5.14}$$

Now, as $\boldsymbol{\alpha}_1'\boldsymbol{\alpha}_1 = 1$; for the expression to be maximised, λ must be the largest possible eigenvalue, and thus λ_1 is the largest eigenvalue corresponding to the first PC Y_1 which has maximum variance. This procedure is repeated for the second PC Y_2, however under the constraint of being orthogonal to Y_1.

As the PCA is to be conducted on the condition ratings of elements within a bridge, the derived PC coefficients α_j indicate the relationship between the bridge elements and new PC Y_j. In the case of the studied BMSs, where low condition ratings correspond to low damage and high condition ratings correspond to advanced damage, positive values of α_j indicates deterioration, and bridges that score highly in Y_j will be seen to have advanced deterioration in these elements. Conversely, α_j with negative values that score highly will indicate elements that are in good condition. This relationship is also seen where low negative scores indicate the opposite to what high positive scores indicate.

5.5.3 Principal Component Scores

From the previous sections, it can be seen that a PCA identifies latent variables for each of the bridges in the data-set which have factor scores, or PC scores, based on the values of the original variables and the PC coefficients α_{ij} (Abdi and Williams 2010). These scores, computed from Equation 5.1, highlight the bridges that most conform to the identified latent variables. In the studied data-sets, each bridge had six condition ratings associated with each element of the bridge. Under the PCA, each of these bridges will now be described by the number of PCs that are chosen to be retained in the analysis; optimally chosen to be less than six, in this case. The scores that each bridge now has will be a function of the original condition ratings and the derived coefficients α_{ij}, as per Equation 5.1. As the magnitude and directions of the vector $\boldsymbol{\alpha}_i$ detail the interrelationships between the bridge elements, the scores obtained for the new PCs Y_i will detail the extent to which the original condition ratings comply with the new latent variable.

5.6 Conclusions

In this chapter, a background has been presented on the use of condition rating data in modern BMSs, obtained through visual inspection. The role that these ratings play in maintenance management was discussed and the potential for using the vast amount of information generated from these inspections in reducing uncertainty in maintenance management was theorised. Methods from which to explore these large data-sets reside in the domain of multivariate data analysis, with a specific emphasis on reconstituting the data in a condensed space through dimensionality reduction techniques. The two methods of interest in this chapter were principal component analysis (PCA) and exploratory factor analysis (EFA). In a comparison of the complexities of the methods, it was shown that PCA was a preferred method based on its absence of subjectivity in the input model, of which is required in EFA. Two data-sets from modern BMSs were presented for analysis, and the level of data required for a stable PCA was determined through Monte Carlo Simulation. It was shown that the level of data available in both data-sets was adequate for a reasonable analysis. The form of output received from a PCA and its interpretation was discussed, and the results of the analysis on the available data will be presented in the following chapter.

Chapter 6

Application of Multivariate Techniques to Bridge Management

6.1 Introduction

In the previous chapter, it was shown how the vast amount of information available on bridge network condition ratings can be exploited through multivariate techniques. For large networks typical of national infrastructure networks, it becomes difficult to parse through this data with any degree of accuracy or reliability, and in this regard, these techniques become necessary to appropriately assess available data and derive more informed condition models from which to make network decisions. By using data reduction techniques, it is possible to define latent variables as functions of the original data-set, which, in this case, highlights common variances within the condition ratings of bridge stock contained in these management systems. A demonstration of the use of PCA will be shown on BMSs from Ireland and Portugal, which shows the utility of reconstituting the large data-sets in a reduced space. Additionally, it will be shown how the newly derived latent variables can provide improved models from which to reduce the uncertainty in maintenance planning based on visual inspection. While the use of this latent variable approach can be used independently of other bridge evaluation techniques, these methods are best used in cooperation with each other. In this regard, this chapter will explore potential future applications of these derived latent variable models in bridge management.

6.2 Region Variant Assessment

A region variant PCA was carried out on condition rating data of masonry arch bridges from BMSs in Ireland and Portugal. Both BMSs operated under the same general ratings system described in Table 5.1, and thus allowed for a direct comparison between the data-sets. The methodology behind conducting the assessments was presented in the previous chapter.

In regards to determining how many PCs to analyse, the Pareto chart is used to combine the rules defined in Section 5.5.1. As can be seen in Figure 6.1, the first PC Y_1 for both data-sets has more than twice the eigenvalue λ of the next PC Y_2. The subsequent PCs have eigenvalues decreasing at a much slower rate, and are close in magnitude, making accurate interpretation of an elbow point the scree line difficult. Using the *Kaiser rule* (Kaiser 1960), it can be seen that just two PCs for each data-set have a value of λ greater than one. However, it can be seen that the retention of the first two PCs Y_1 and Y_2 account for 53.3% and 57.1% of the Irish and Portuguese data-sets, respectively. This would fall short of the cumulative variation threshold, which looks for 70%–90% of variation to be retained. By relaxing the *Kaiser rule* as per Jolliffe (1972), it can be seen that there is scope to retain a further two PCs in analysis. By retaining three PCs, 68.6% and 71.1% of the variance of the variance is accounted for, while retaining four PCs would keep 82.3% and 81.8% of the variance in the two data-sets. Additionally, it is obvious that Y_1 accounts for a significant amount of the variation here; totalling approximately 37% and 40% of the Irish and Portuguese data-sets, respectively. Thus, Y_1 here is deemed to be the primary PC for these networks.

For Y_1, it can be seen that each element has a positive value for α_1, and it can also be seen that there is some correlation between the two data-sets (Figure 6.2). As each α_1 is positive, it can be said that Y_1 is a latent variable from the data-sets that describes the general state of deterioration of a bridge within the data-sets; where a high positive score indicates advanced damage for all the elements in the bridge, and a low negative score indicates bridges where these elements are in favourable conditions. In fact, as the largest coefficients are for the primary structural elements, Y_1 can further be described as a measure of the structural condition of the bridge; as these coefficients have the largest influence on Y_1, and thus prioritise the condition in these elements over that of the minor elements.

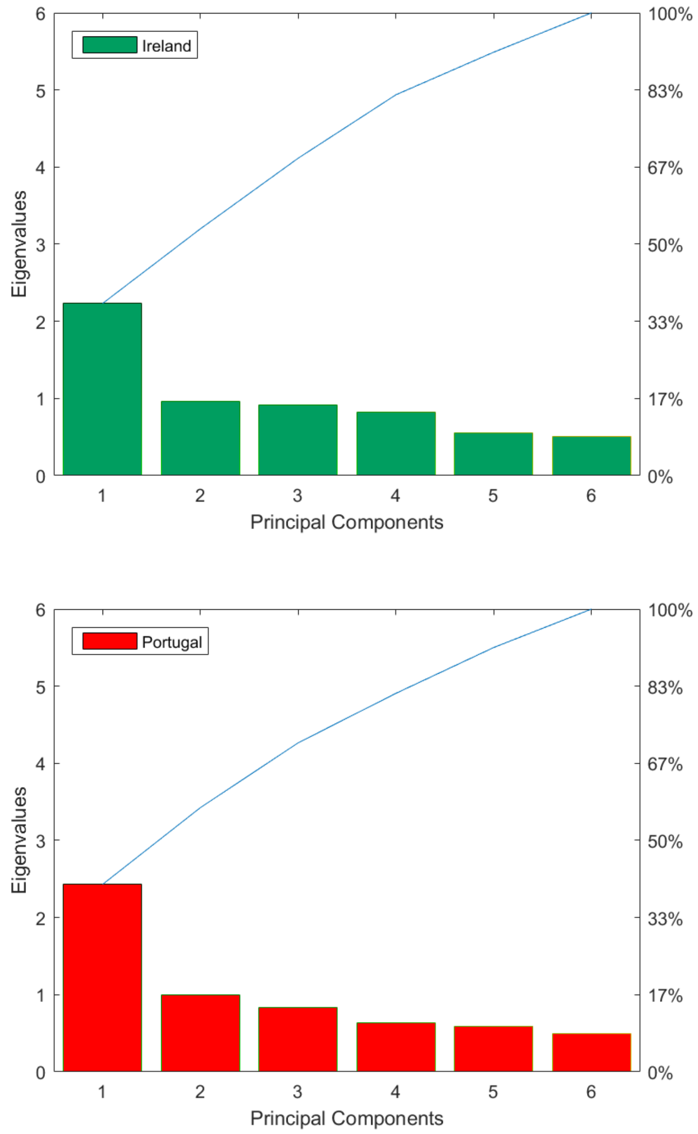

Figure 6.1: Pareto plot of principal components for Irish (a) and Portuguese (b) bridges in the data-sets.

Unlike Y_1, it can be seen that Y_2 is the latent variable that describes a situation where there is a discrepancy between the condition ratings of the parapets, surface, and embankment, and the ratings for abutments, walls, and deck; or, generally speaking, the non-structural and structural elements (Figure 6.3). Additionally, it can be seen that there is an apparent inverse correlation between the bridge stock of Ireland and Portugal. However, it can be seen that the absolute values of α_2 for each element are very similar across the two countries. However, this inverse relationship can be explained by the number of each bridges in the original data set that exhibited this relationship, where the distribution of condition ratings for abutments, walls, and deck in the Irish data-set contained

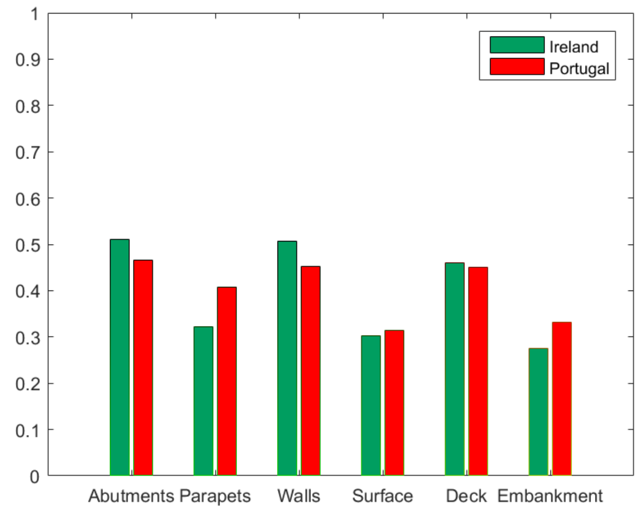

Figure 6.2: Coefficients for the first PC Y_1.

a larger number of higher condition ratings than in the Portuguese data-set; and similarly, it was seen that the Portuguese data-set had larger number of higher condition ratings for the elements parapets, surface, and embankments.

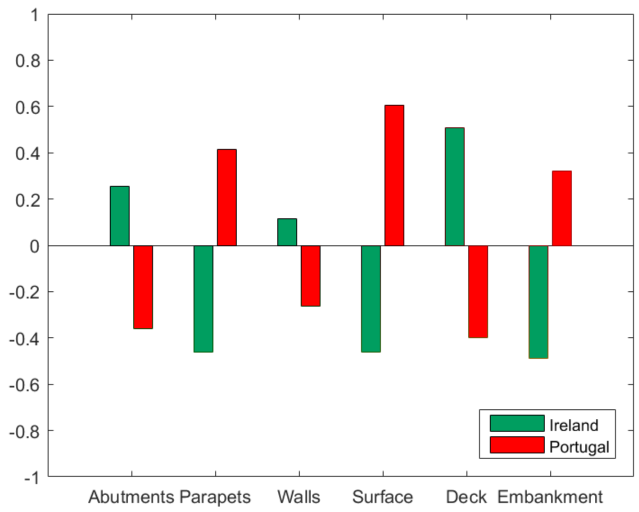

Figure 6.3: Coefficients for the second PC Y_2.

The third PC Y_3 mostly describes bridges that have a discrepancy between the condition ratings of the embankments and surface of the bridge (Figure 6.4). The small PC coefficients α_3 for abutments, walls, and deck show that these elements are not very influential in this PC. However, it can be seen in the Irish data-set that the parapets have significantly more influence on Y_3 that the parapets in the Portuguese data-set. On a reduced scale, the opposite is true for the deck element where, in the Portuguese data-set, this element has more influence than in the analysis for the Irish data-set.

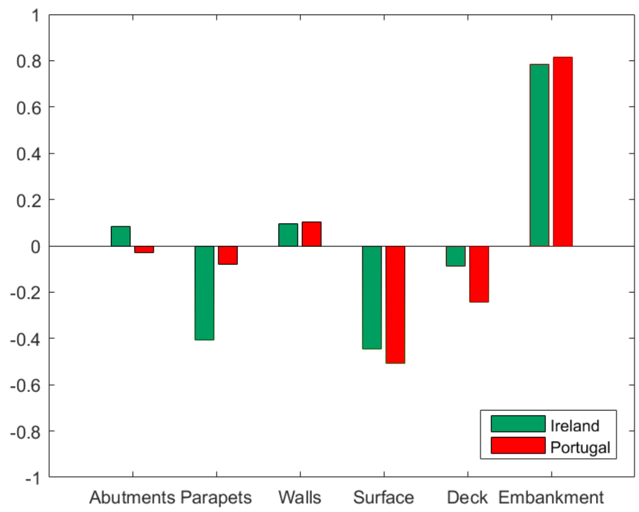

Figure 6.4: Coefficients for the third PC Y_3.

The fourth PC Y_4, similarly to Y_2, demonstrates an apparent anti-correlation in α_4 between the elements in the Irish and Portuguese data-sets (Figure 6.5). Again, however, the absolute magnitudes of these coefficients are quite similar, and typical of the relationship seen so far for the first three PCs. This PC describes bridges that have a disparity in condition ratings between the parapets and the abutments, with the other elements not proving to be very influential. Thus, Y_4 can be said to represent the 'topside' of a bridge, also the aspects of the bridge most encountered by bridge users.

A further comparison between the two data-sets can be observed by squaring the PC coefficients α_{ij}, which offers a direct comparison between the absolute values of α_{ij} for each element (Tables 6.1 & 6.2). This provides a different comparison than presented previously, in that for Figures 6.2 and 6.3, the direction of α_{ij} (i.e. positive or negative) gave an indication to the relative number of bridges with elements in a state of deterioration above or below average for the data-set. This is most evident in Figure 6.3, where α_{ij} for Ireland and Portugal have opposing directions but similar magnitudes. By comparing the α_{ij}^2, it is possible to compare the relative contributions of the original variables to the new latent variables; in this case being Y_2. By plotting the α_{ij}^2 of the Irish data-set against that of the Portuguese data-set, it can be seen that there is a good degree of linear correlation between the two, yielding a coefficient of determination R^2 of 0.8605 (Figure 6.6). However, it can be seen that the linear

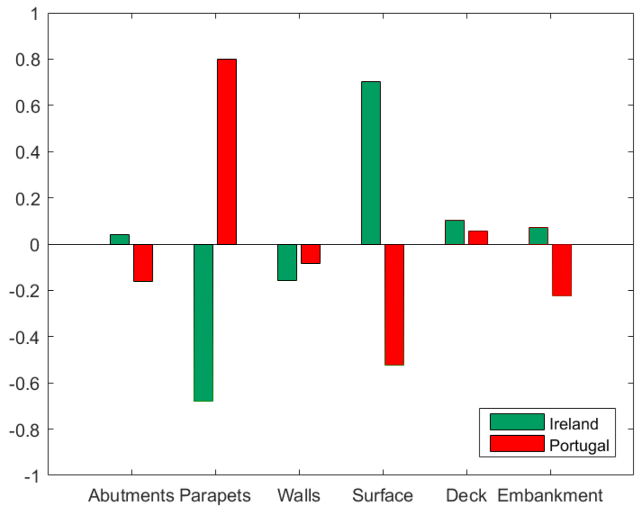

Figure 6.5: Coefficients for the fourth PC Y_4.

fit tends towards the Irish axis, suggesting an unbalanced comparison between the two data-sets. Such a relationship, in addition to differences seen in the magnitudes of the α_{ij} show that any attempt to compare condition data from similar but geographically different BMSs requires the consideration of local variances within the data, with a calibration of these variances being necessary for direct comparison or homogenisation of the systems.

Table 6.1: Squared coefficients for first and second PC, α^2_{1i} and α^2_{2i}

	Y_1		Y_2	
Element	Ire	Por	Ire	Por
Abutments	0.2605	0.2166	0.0651	0.1296
Parapets	0.1035	0.1669	0.2141	0.1710
Walls	0.2575	0.2043	0.0135	0.0698
Surface	0.0918	0.0989	0.2131	0.3675
Deck	0.2112	0.2033	0.2573	0.1584
Embankment	0.0755	0.1099	0.2368	0.1037
Σ	1.0000	1.0000	1.0000	1.0000

It has been shown that Y_1 is a measure of the overall condition of the bridges, being a linear combination of the deteriorated state of each element. Thus, bridges that have high positive scores for Y_1 are expected to have damage present in each element, and bridges with low scores should generally be in a favourable state. This was confirmed by investigating the original data-sets, which showed that the bridges with the lowest scores had condition ratings of 0 for each element, and the bridges with the highest score typically showed dam-

Table 6.2: Squared coefficients for third and fourth PC, α_{3i}^2 and α_{4i}^2

	Y_3		Y_4	
Element	Ire	Por	Ire	Por
Abutments	0.0068	0.0008	0.0015	0.0264
Parapets	0.1658	0.0066	0.4634	0.6402
Walls	0.0088	0.0103	0.0253	0.0068
Surface	0.1968	0.2595	0.4941	0.2726
Deck	0.0074	0.0592	0.0107	0.0031
Embankment	0.6143	0.6637	0.0050	0.0508
Σ	1.0000	1.0000	1.0000	1.0000

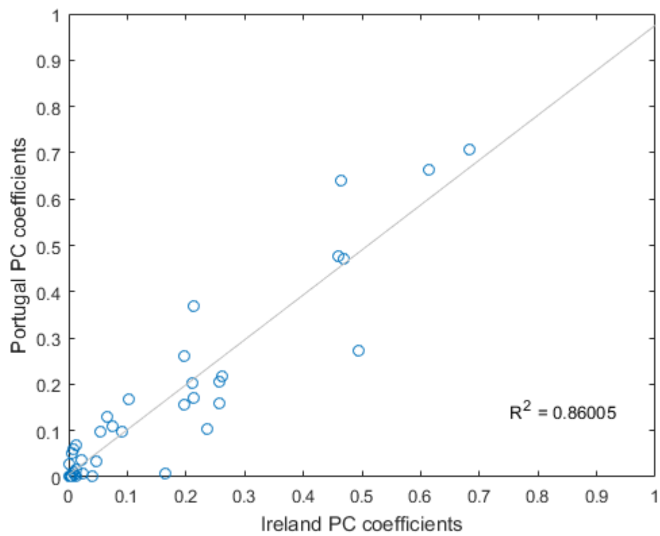

Figure 6.6: Relationship of α_{ij}^2 for Irish and Portuguese data-sets.

age present in each element, with advanced condition ratings for each element typically between 3 and 5. Similarly, it was seen that bridges that had extreme scores for Y_2 were those that exhibited a discrepancy between the elemental condition ratings of the structural and non-structural elements. As using either model for Y_2 on both data-sets would significantly skew the results for the other data-set, the widespread application of the latent variable model for PCs with directional disagreement requires further refinement. However, for PCs in general directional agreement, such as Y_1, it is possible to apply a single model across both data-sets; providing the magnitudes of α_{ij} and α_{ij}^2 are in general agreement.

In this analysis, it has been shown that there is some variation in same-structure values for α_{ij}^2 between Ireland and Portugal; initially suggesting that a homogenised, region-invariant latent variable approach cannot be employed across differing BMSs. However, when comparing the condition ratings from both data-sets using the first PC models for Ireland ($Y_{1,Ire}$) and Portugal ($Y_{1,Por}$), it can be seen that there is considerable agreement between the overall condition under both models (Figure 6.7).

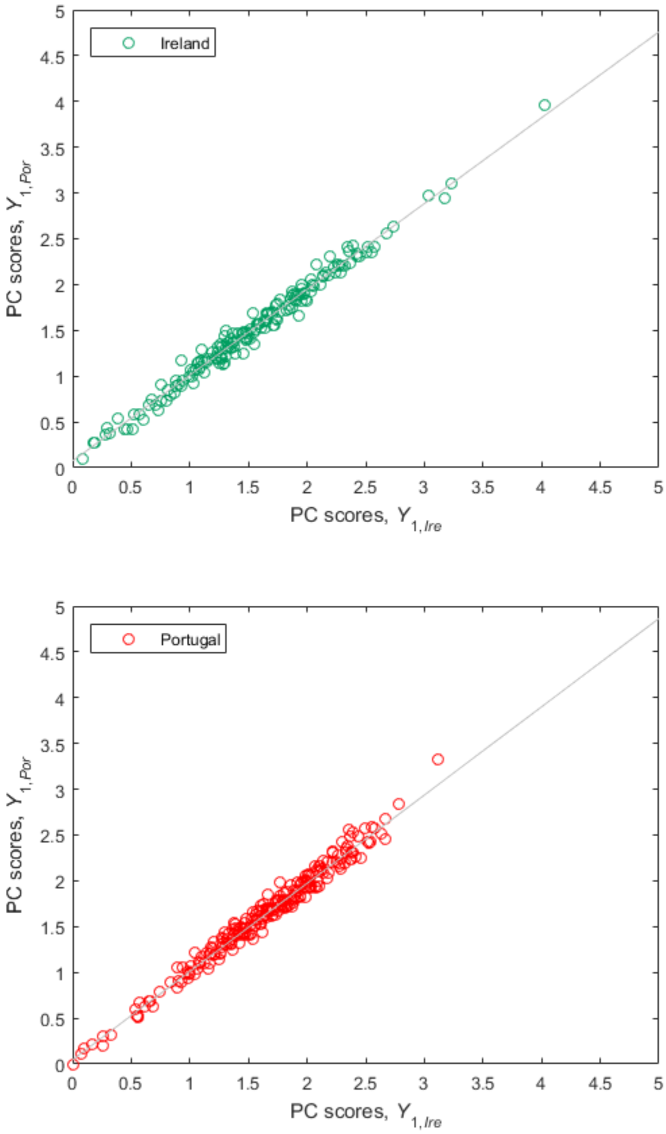

Figure 6.7: Comparison of condition rating for Ireland (a) and Portugal (b) under different regional models for Y_1.

The linear relationship between these two models is seen to be high, exhibiting an R^2 of 0.98 for both comparisons. Similar to Figure 6.6, it can be seen that the relationships tend toward the Irish axis, suggesting that the model for $Y_{1,Ire}$ derived from the Irish data is slightly more conservative than that for the Portuguese model $Y_{1,Por}$. As this conservatism is based on a model derived from existing data, it is necessary to explore a larger data-set from more regions to establish if a homogenised model can be developed to describe the overall condition of specific structures, or if regional variances must be accounted for to result in a homogenised description of the overall damage state.

6.3 Structure Variant Assessment

To explore a PCA on a larger data-set, it was decided to study the Portuguese data-set in more detail, as it contained 3,036 bridges in total. The most numerous of these bridges were reinforced concrete (1690) and masonry arch (713) bridges. The methodology used in the previous analysis is repeated here, in that the data-set analyses consisted of condition ratings on six elements, as this allowed the largest analysis.

From Figure 6.8, it can be seen that the PC at which the plot begins to flatten out, or the elbow, occurs for both bridge types at the third PC, Y_3. The first three PCs for the reinforced concrete and masonry arch bridges account for 74% and 71% of the variation in the data, respectively. Additionally, these three PCs also satisfy the established relaxation of the *Kaiser rule*. Including the fourth PC results in retaining 84% and 82% of the variation, but this PC can be discounted based on the established retention criteria.

From these plots, it is clearly evident that the first PC Y_1 retains the most significant amount of variation; accounting for 41% of the variation in both bridge types, and is thus the primary PC. For Y_1, it can be seen that each element has a positive value for α_1, and it can also be seen that there is some correlation between the two data-sets (Figure 6.9). As each α_1 is positive, it can be said that Y_1 describes the general state of deterioration of a bridge in the data-sets, where a high positive score indicates advanced damage for all the elements in the bridge, and a low negative score indicates bridges where these elements are in favourable conditions. In fact, as the largest coefficients are for the primary structural elements, Y_1 can further be described as a measure of the structural condition of the bridge.

Notably, it can also be seen that there is some correlation between the two data-sets, despite bridges having different structural forms and being constructed with different materials. It can be seen that the greatest deviation occurs for the embankment, where α_1 for this element is less influential in the reinforced concrete bridges than in the masonry arch bridges. This demonstrates, in a

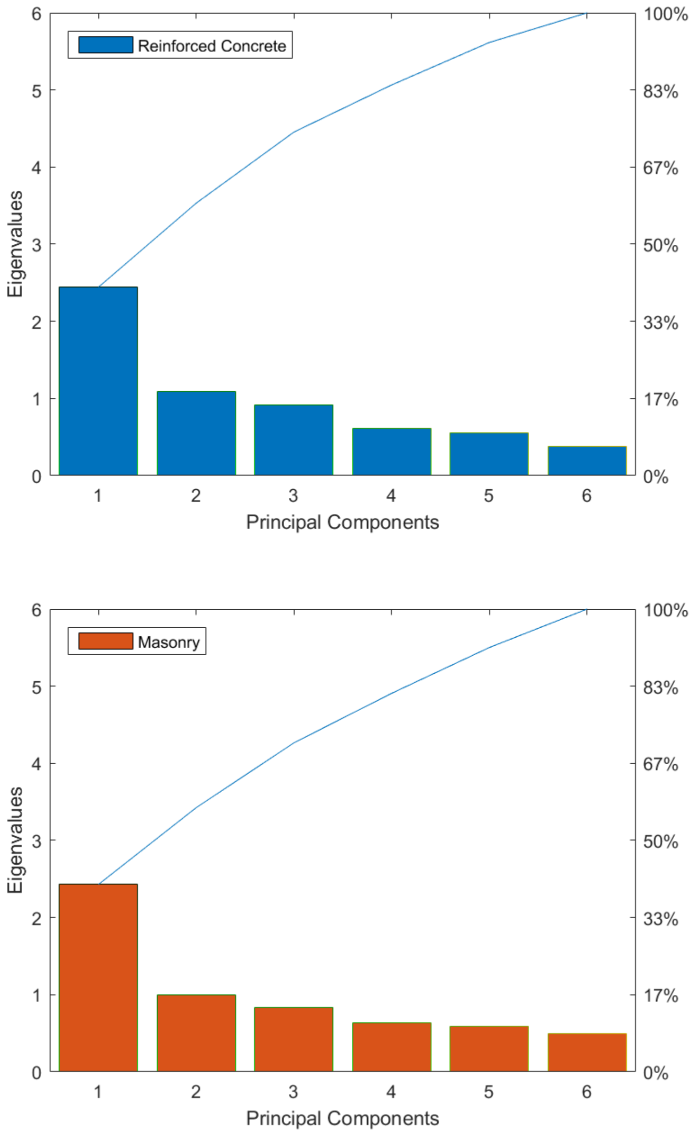

Figure 6.8: Pareto plot of principal components for reinforced concrete (a) and masonry arch (b) bridges in the data-sets.

way to be expected, that while many elements will behave in a similar way regardless of the bridge type, there remains a number of elements that are specific to certain bridge types, and exercise their own degree of influence on the PCA accordingly. This suggests that the PCA method needs to be applied in a more targeted fashion, and should not be inappropriately applied to an entire data-set of a BMS, if the population of bridges is non-uniform. This provides opportunities to cluster or bunch the data based on associated meta-data, and establish defined signatures for various bridge types.

Unlike Y_1, it can be see that Y_2 describes a situation where there is a discrepancy

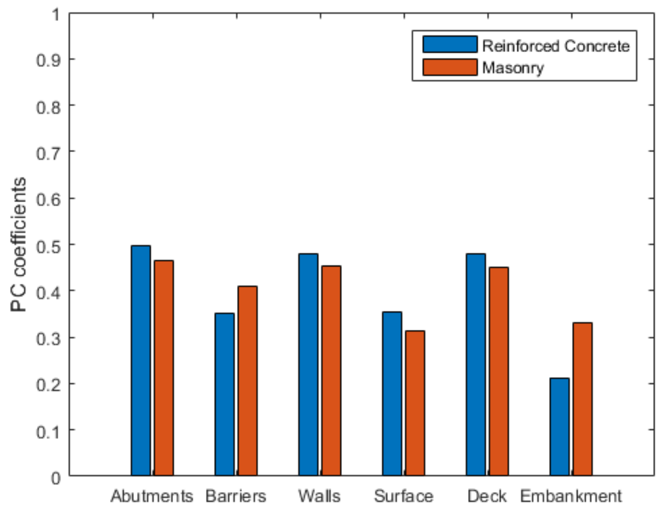

Figure 6.9: PC coefficients for the first PC, Y_1.

between the condition ratings of the parapets, surface, and embankment, and the ratings for abutments, walls, and deck, or, simply, Y_2 can be said to describe bridges where there is a disparity between the condition ratings of the structural and non-structural elements (Figure 6.10). This would suggest that there are a greater proportion of bridges in both data-sets that have structural elements in good conditions where non-structural elements had exhibited damage. This is often typical of asset-management strategies for bridges where the structural elements are subject to a greater repair priority than the non-structural elements.

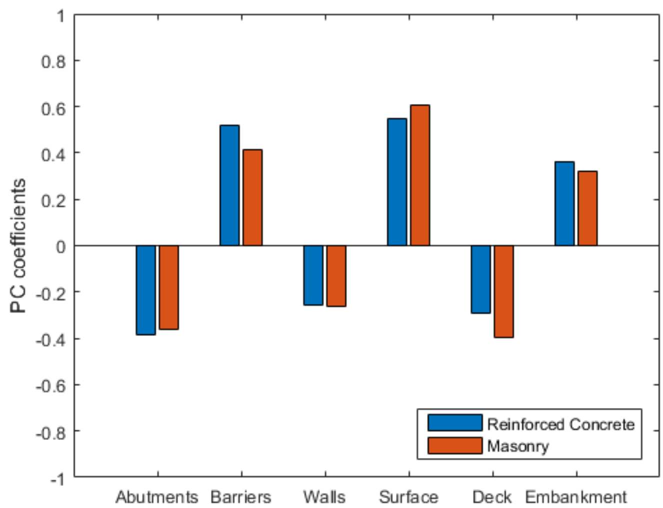

Figure 6.10: PC coefficients for the second PC, Y_2.

The third PC Y_3 mostly describes bridges that have a discrepancy between the condition ratings of the embankments and surface of the bridge (Figure 6.11). The small PC coefficients α_3 for abutments, walls, and deck show that these elements are not very influential in this PC, and thus it can be said that this PC primarily is a measure of the condition of the embankment and its relationship to the condition of the surface. Here, however, we see some deviation based on bridge types, where for masonry arch bridges the surface is the second most influential element, whereas for reinforced concrete bridges this influence is attributed to the barriers and thirdly the surface. This can be explained by how reinforced concrete bridges are likely to be more modern than masonry arch bridges, and are thus more likely to have traffic barriers installed, in addition to the parapets. The structural elements of abutments, walls, and deck account for little influence in this PC.

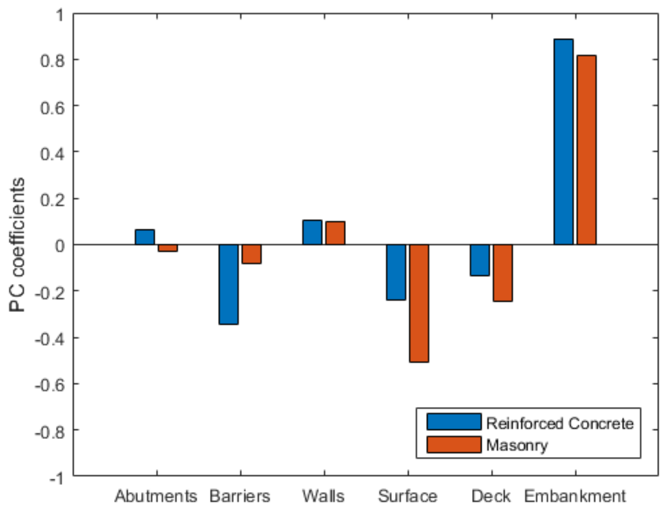

Figure 6.11: PC coefficients for the third PC, Y_3.

It has been shown that Y_1 is a measure of the overall condition of the bridges, being the deteriorated state of each element. Now, as Y_1 shows the overall damage of the bridge, by way of the amount of damage in each element, these scores can be compared to the overall condition rating for each data-set. However, it can be seen that there does not exist a high correlation between these overall condition ratings and the PC scores. This can be seen for the reinforced concrete bridges in the data-set (Figure 6.12) and the masonry arch bridges in the data-set (Figure 6.13). For both these data-sets, the coefficient of determination (R-squared) is approximately 0.6.

From these figures, it is obvious that there is a significant discrepancy between this overall condition rating and Y_1, which appears to represent the overall state

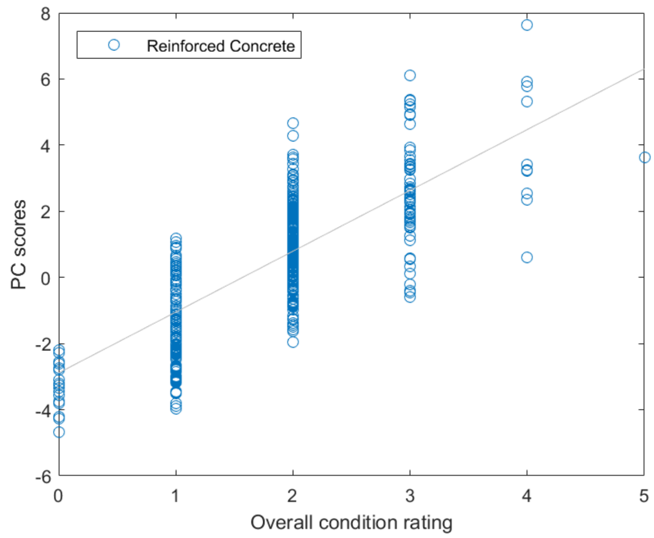

Figure 6.12: Correlation between PC scores for Y_1 and the overall condition rating (reinforced concrete).

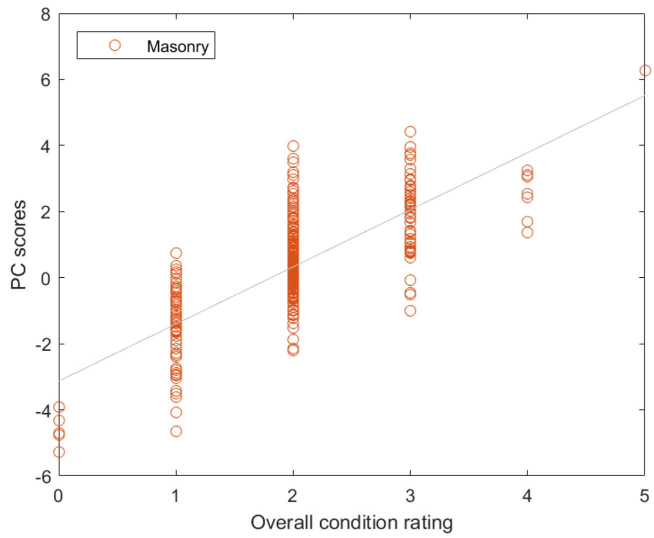

Figure 6.13: Correlation between PC scores for Y_1 and the overall condition rating (masonry arch).

of the structure. There exists an overlap of condition ratings for the same scores, with some bridges on the same score having condition ratings of 1, 2, and 3, for example, when their elements are largely in the same condition.

6.4 Integration to Existing BMS

From the previous section, it is clear that there is scope for an improved form of the overall condition rating, in the guise of a linear combination of the condition ratings of the bridges individual elements. However, as some elements are more important to the structural condition, an equal weighting should not be applied to each rating. In regard to Equation 5.2, it is seen that the sum of the square of the PC coefficients α_i equal unity, and thus α_i^2 for each variable quantify its relative influence and importance in the PC score. As the score for Y_1 has been shown to be a good descriptor of the overall condition of the bridge, the weighting factors can be derived from the squared coefficients α_i^2 as a first step. In this regard, the revised weighted condition rating ζ of the overall structure can be defined as:

$$\zeta = \sum_{j=1}^{p} \psi_j x_j, \quad \psi_j = \alpha_{1,j}^2 \tag{6.1}$$

Where ζ is a linear combination of the new weighting factors ψ_j and the original condition ratings x_j for the individual elements. The new weighting factors ψ_j for each element for both bridge types can be seen in Table 6.3.

Table 6.3: Weighting factors, ψ

Element	Reinforced concrete	Masonry arch
Abutments	0.2466	0.2166
Barriers	0.1238	0.1669
Walls	0.2290	0.2043
Surface	0.1255	0.0989
Deck	0.2300	0.2033
Embankment	0.0451	0.1099
Σ	1.0000	1.0000

From this, it is clear that ζ is weighted further towards the structural elements than the non-structural elements, and would align with typical positions adopted by bridge managers.

The approach of using weighting factors to determine a more realistic interpretation of the overall state of the bridge stock allows for a simple integration into an existing BMS, and does not require formalising a multivariate procedure into an existing framework. This is demonstrated for the presented data-sets, whereby Equation 6.1 was applied to the original condition rating data and compared the PC scores for Y_1. It can be seen for the reinforced concrete (Figure 6.14) and masonry arch (Figure 6.15) bridges that there is high correlation between the PC scores and the revised weighted condition rating ζ. Additionally, ζ is presented in the same condition rating range as the original data-set, which allows for the simple comparison between ζ and the existing overall condition rating. While the current overall condition rating is useful in determining the need for intervention, as an overall rating above a defined threshold triggers action, its use in describing the state of the complete asset base can result in a significantly misleading assessment. Additionally, the proposed weighted condition rating can be used in conjunction with the existing overall condition rating, whereby when presented with a high number of bridges which call for immediate intervention, ζ can be used to prioritise which bridges are most in danger structurally. This provides an extra decision tool that can be used, in addition to such aspects as the bridges importance to the road network, as well as cost of intervention activity.

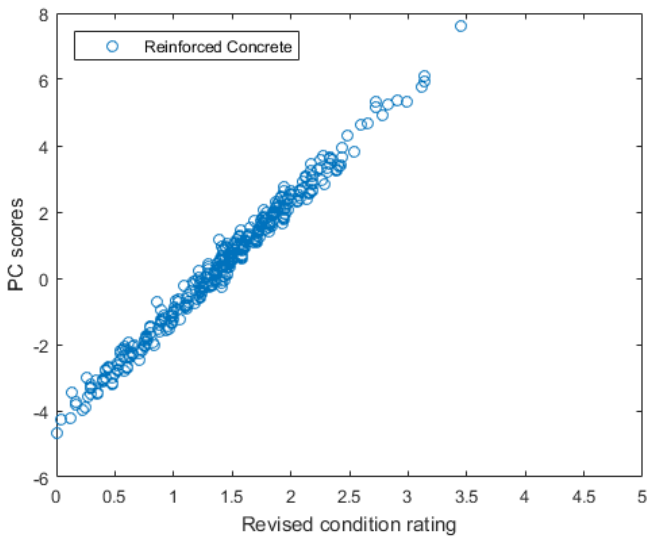

Figure 6.14: Revised weighted condition rating against Y_1 for reinforced concrete bridges.

In regards to establishing the current damage state of the entire bridge inventory, using these revised ratings can allow for the establishment of a more refined distribution based on a histogram of smaller intervals. For the original condition ratings, any distribution for the bridge inventory must be derived from a histogram of 6 discrete intervals of $0 \rightarrow 5$. However, under the proposed model, there is a greater spread in the frequencies of the condition ratings, leading to a smoother distribution. As ζ is a function of the PC scores Y_1, it is prudent to first consider the distribution of scores for this PC when looking at the inventory. From Figures 6.16 and 6.17, it can be seen that the scores

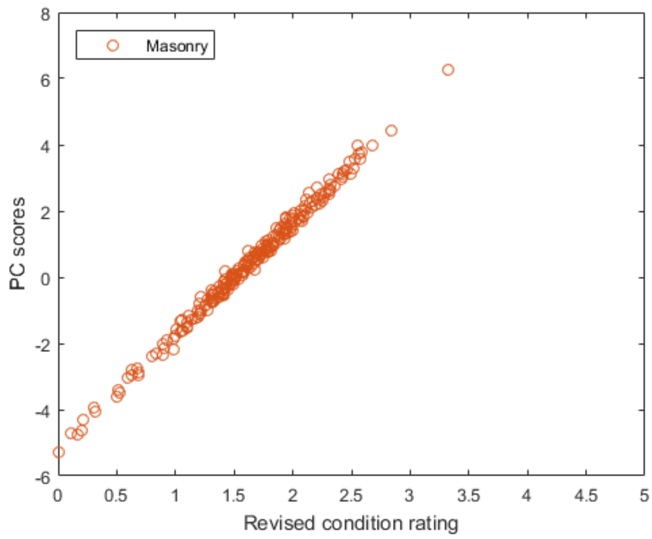

Figure 6.15: Revised weighted condition rating against Y_1 for masonry arch bridges.

for the first PC generally conform to a normal distribution. This is expected as these scores are contained in the positive and negative domain, and are thus compatible with a normal distribution.

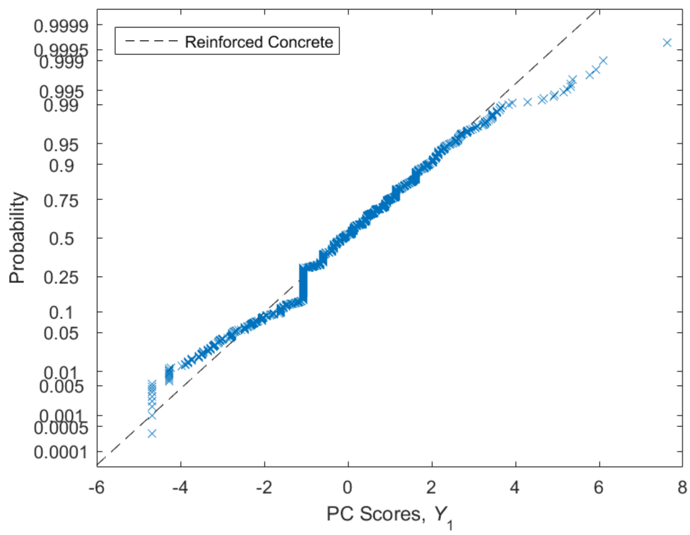

Figure 6.16: Normal probability plot for the distribution of the scores for Y_1 for reinforced concrete bridges.

However, when reconstituted into ζ, which is exclusively positive in the presented model, it is not appropriate to model the data as a normal distribution. For this reason, a lognormal distribution was fitted to represent the revised condition ratings. From these distributions, it can be seen that the mean condition

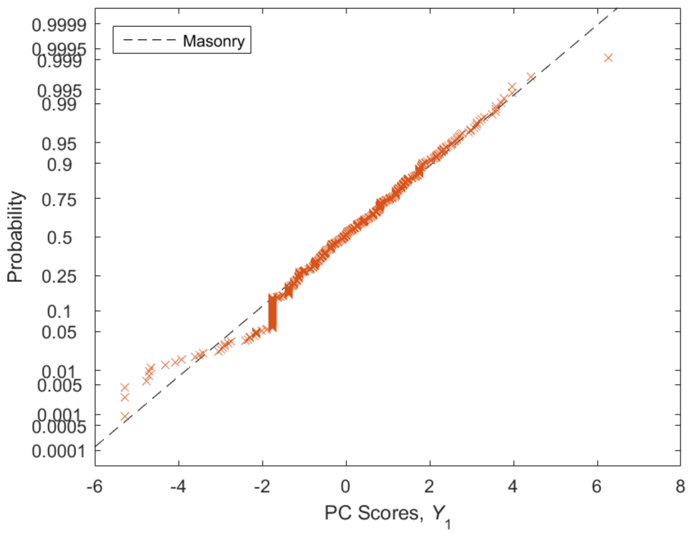

Figure 6.17: Normal probability plot for the distribution of the scores for Y_1 for masonry arch bridges.

rating is 1.36 for reinforced concrete bridges and 1.54 for masonry arch bridges from the Portuguese data-set (Figure 6.18).

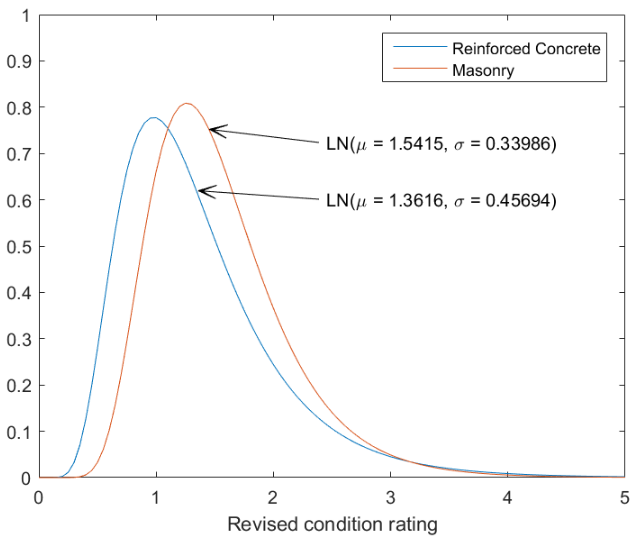

Figure 6.18: Distribution of revised weighted condition ratings for reinforced concrete and masonry arch bridges.

Being able to classify certain bridge types within an inventory in this fashion allows for significant planning in regards to future intervention strategies, and the expected performance of a network. Using this approach to condition ratings can help identify at-risk bridges and bridge types, and allows greater forward planning in regards to future maintenance needs.

From this analysis, it was seen that the PC coefficients represented a good starting point where to establish weighting factors ψ_j to existing condition ratings, to create a revised weighted condition rating ζ. As a first point, these weighting factors can be presented to asset-managers for their input, and determine if these factors can be refined based off the experience of asset-managers and their decision making tools. In order to present these first point weighting factors, it will be necessary to investigate further bridge types, in order to see if generic weighting factors are applicable, or if it is necessary to have bespoke factors based on the structural form and location of the bridge. While this analysis was conducted on two BMSs that operated under the same general framework, it should be possible to use the PCA method to compare bridge stocks assessed under various BMSs, as an underlying latent variable for the general condition can be found for a direct comparison. Gaining access to more data-sets of national bridge inventory is a priority in this regard so that a definitive condition assessment model can be developed; and should regional variances become a significant factor, the development of region-specific adjustments factors can be explored to align regions under a single latent variable model for condition assessment. It can be seen that this chapter addresses the final research Objectives, 4 and 5, outlined in Chapter 1.

6.5 Conclusions

A PCA was conducted on data-sets of masonry arch bridges in Ireland and Portugal, as well as reinforced concrete bridges in Portugal, to exploit dimensionality reduction techniques to establish latent variables. It was seen that there was good correlation in the PCs, where elements typically had the same influence on the PC across the two countries and both bridge types. Despite some directional variances in the PC coefficients α_{ij} between the two data-sets, it was observed that the absolute and squared values of α_{ij} were in general agreement across both regions. The variances in direction for some values of α_{ij} can be attributed to the percentage of bridges within each data-set that had elements in a favourable or unfavourable state. This demonstrates how PCA can be an effective tool when looking for a direct comparison between the relative health or condition of elements in multiple data-sets; as an agreement in magnitude implies reliability in the model, and disagreement in direction indicates how elements in one data-set are performing against the other.

Further to this, beyond using data-reduction for comparative purposes, the possibility of developing a homogenised, region-invariant condition rating model through latent variables can be seen in this chapter; based on the models for Y_1 derived from both data-sets. As a first step, these models can be the foundation from which to build on a more reliable condition assessment model; which can be refined based on input from experienced bridge managers and through further analysis on a wider array of data from different regions and for more bridge types. By using the vast amounts of data currently available but sometimes being under-utilised, it is possible to better inform decision making by using this data to reduce uncertainty levels surrounding maintenance activities dictated by condition rating data.

Chapter 7

Conclusions

7.1 Summary of Research

This thesis focused on the effects of disparate information levels on bridge management and safety. The work focused on variable levels at which information is available for a bridge network. Structural reliability is the most sophisticated assessment method and allows for structure specific analysis, but can be intensive in its requirement for information gathering, and so information relating to reliability assessment is not plentiful. On the other hand, visual inspection of bridges, with its limitation in terms of actual capacity information, is available for thousands of bridges in many networks. By evaluating these methods, it was seen that there was a degree of subjectivity to how they are conducted and this affects the results that they garner.

It was observed that uncertainty in information around resistance and load variables affects the estimates of reliability and condition evaluation. By looking at different levels of uncertainty in structural reliability analysis, it was possible to extract bridge-specific information on important aspects from which to monitor and gain further information. From the perspective of loading, it was seen that bridges designed and constructed in specific eras can be more susceptible to limit-state violation based on the design load models used at the time.

For the condition rating data, it was seen that there is a discrepancy between overall condition ratings, that are not assigned as a function of element condition ratings, and latent variables, that work as a linear combination of these elemental ratings. These latent variables were seen to provide a better indicator for bridge condition, and can be used to improve intervention decisions for bridge managers and stakeholders.

7.2 Detailed Results

The objective of this thesis was to investigate the effect of disparate information levels on bridge management and safety, and how the levels of information available to the engineer can severely impact the stability of probabilistic assessment. Five specific objectives were outlined in Chapter 1, and they were addressed as follows:

1. In Chapter 3, a structural reliability analysis was conducted on three bridge types typical of Ireland and mainland Europe; all of which were assessed for the limit-states of flexure under a probabilistically defined load model. The emphasis on this work was regarding the levels of uncertainty in the model parameters of the physical nature of the bridges, and how these affected different bridge types. It was observed that bridges of similar structural material and form are clustered in terms of sensitivity or parametric importance studies. The levels of existing correlations for the parameters across the bridge types, and how their influences on the reliability under varying degrees of uncertainty indicates the importance of a calibrated framework for the assessment of bridges at a network level. The network-level calibration is observed to be strongly dependent on the availability and the quality of information of the bridges within the network and, consequently, it can be stated that structural reliability analysis refers more to our state of knowledge of the structure than to the actual state of the structure itself. This emphasizes the need for data-sharing for such structures by the managers and owners of bridge networks for the most reasonable and cost-effective interventions to be carried out.

2-3. In exploring the effect of uncertainty surrounding load models in assessment, a structural reliability analysis was conducted on three bridges in Chapter 4 to assess the effect of changing definitions of code-defined traffic loading on safety classifications of the structures. It was observed that earlier codes produced less onerous flexural load effects and, as such, resulted in a reduced demand for flexural capacity and thus reliability indices closer to that determined under the probabilistic load model; making them more susceptible to limit-state violation and a greater intervention burden. It was shown that bridges produced under loading prescribed by modern standards produced bridges with a higher β assessed under a probabilistic load model, and resulted in a significantly reduced expected life-cycle cost; despite the increased initial construction costs due to a higher minimum requirement for flexural reinforcement. This increased initial cost was seen to be significantly offset with a lower expected cost of failure over the bridges life-cycle. Given the disparity between β for the probabilistic load model and the more recent codes of practice, it is evident that, while bridge structures designed and constructed according to these standards should have a higher resistance capacity than seen in bridges designed to the extent of the earlier standards, the use of the load model itself for assessment does not reflect the true operating state of the bridge.

4. The prevalence of visual inspection based condition ratings in bridge maintenance management showed the potential for using the vast amount of information generated from these inspections to reduce network uncertainty by extracting patterns from a large data-set. In Chapter 5, methods from which to explore these large data-sets were shown to be *principal component analysis* (PCA) and *exploratory factor analysis* (EFA); both data reduction techniques to explore maximum variance in a data-set. In a comparison of the complexities of the methods, it was shown that PCA was a preferred method based on its absence of subjectivity in the input model, and it was shown that the level of data available for analysis was adequate for a reasonable assessment. In Chapter 6, the analysis was conducted in on data-sets of masonry arch bridges in Ireland and Portugal, as well as reinforced concrete bridges in Portugal, to exploit dimensionality reduction techniques to establish latent variables. It was seen that there was good correlation in the PCs, where elements typically had the same influence on the PC across the two countries and both bridge types. Despite some directional variances in the PC coefficients α_{ij} between the two data-sets, it was observed that the absolute and squared values of α_{ij} were in general agreement across both regions. This demonstrates how PCA can be an effective tool when looking for a direct comparison between the relative health or condition of elements in multiple data-sets.

5. In Chapter 6, it was observed that the analysis derived a latent variable (PC) that provided an indicator of the overall condition of the bridge

based on the conditions of the individual elements, and it was notable that when plotted against the overall condition ratings of the bridges, there was little correlation observed with the recorded values in the data-set; showing the subjectivity of overall condition rating scores assigned semi-independent of the condition of the bridges elements. Referring back to the original data-set and ranking the bridges according to their scores for this PC, it was seen that bridges with the lowest scores had elements in favourable conditions, while the bridges with the highest score showed advanced damage in each element. A new condition rating model based on weighting functions was proposed based on this latent variable, which was seen to perform at a higher level than existing overall structure classifications based on condition ratings. In addition, it was possible to create a refined distribution of the condition ratings for specific bridge types; which can enable an improved ranking of bridges in the network and better highlight at-risk bridges. By using the vast amounts of data currently available this way, it is possible to better inform decision making by using this data to reduce uncertainty levels surrounding maintenance activities dictated by condition rating data.

7.3 Critical Assessment of Developed Work

As this thesis is predicated on the availability and use of information in bridge management and safety, so too is this thesis affected by the availability of this data.

For the analysis presented in Chapters 3 and 4, the assessment was constrained by the availability of probabilistic loading information, and so the span length was fixed for the three bridges used in the analysis. The use of site-specific loading data on a larger number of bridges with varying geometry would result in a more definitive conclusion. As the superstructure of the bridges varied between reinforced and prestressed concrete slabs and beams, further study is recommended on additional bridge types and for varying span lengths before a definitive outcome can be reached. The use of typical bridge geometry and random variable distributions, as opposed to real bridges and actual material property information, produces nominal results; but are adequate from which to draw a general conclusion. Although the life-cycle cost analysis produced encouraging results, further questions remain as to whether there is an optimum point at which the initial cost can be increased to minimise the total expected life-cycle cost, and whether there are further variables that can be optimised at the design stage for bridges. Additionally, the use of other life-cycle cost models would allow for a greater degree of benchmarking from which a more accurate forecasting of life-cycle costs at this design stage. Furthermore, the practical ways such a philosophy can be adopted in codes of practice are unanswered; be it through refinements of partial factors for resistance variables such as the area of steel A_s or the compressive strength of concrete f_c, or through a more holistic increase in safety factors regarding the applied traffic loading. Despite these remaining issues, it is clear from the presented results that there is scope for significant savings through a more conservative approach at the design stage.

While the PCA presented in Chapters 5 and 6 was conducted on data-sets of bridges from Ireland and Portugal and a direct comparison between the output for each analysis was made, the regional effects on the data-sets was not explored due to the lack of information available on maintenance practices or environmental factors; due to confidential nature of the data received. The work presented showed good utility as a way to compare data-sets, but more holistic information needs to be accounted for in order to achieve a more refined comparison. Additionally, efforts should be made to determine the age of the bridges under assessment, as this would allow the for time-varying comparisons between bridges under different stages of their life-cycle. However, this data is not always available in BMSs, due to availability of historic records surrounding bridge construction.

7.4 Recommendations for Future Research

Based on the results presented in this thesis, there are a number of further research areas which logically follow; which include, but is not limited to:

- Further analysis into benchmarking the uncertainty in reliability in different bridge types, and establishing parameter sensitivities and importance measures to define bespoke areas of interest for different types of bridge construction
- Detailed life-cycle cost analysis on additional bridge types and using different financial models to establish an optimum level for which to balance construction cost and life-cycle cost. This optimum point could be quan-

tified as a global or partial factor to be applied to newly designed bridges in networks of economic importance, from which to reduced maintenance and intervention burden

- Further PCA on larger data-sets, along with other methods of multivariate analysis under which to establish patterns and clusters in existing networks that can inform decision making. This analysis should incorporate greater investigation into different parameters which form meta-data for the existing condition rating data. These include bridge span, location, environmental factors, etc.
- The integration of this multivariate analysis into a risk-based assessment framework. This can be linked to reliability analysis under a number of ways currently being researched, which includes the use of Bayesian Belief Networks, Bayesian updating, and value of information concepts in reliability

This thesis is presented as a first step from which to exploit the modern landscape of large data-sets being created and stored in BMSs, in an effort to better inform some of the more subjective aspects of bridge management. As these modern BMSs are now widely employed by bridge management authorities, the level of data available will continue to grow, presenting researchers greater opportunity from which to understand networks at a larger level.

While structural reliability and a risk-based approach to bridge management represents the most compelling method for intervention decisions and resource allocation, it must be done in the confines of the existing environment for bridge management, in order to ease a widespread application. In order to accomplish this, condition rating data must be used to inform probabilistic assessments as much as is reasonably practicable. In that regard, efforts must be taken to relate condition rating data with NDE results, so as to inform a probabilistic model with reduced uncertainty into the resistance capacity of the structure. By refining the results obtained from the multivariate assessment, a more definitive and unambiguous condition rating can be obtained for a bridge and its elements, from which it can be possible model the resistance parameters without the necessity for expensive NDE. While the true reliability can only be determined using NDE and site-specific data, a network assessment using such a correlation can be used to identify bridges that require further assessment in this regard.

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Appendix A

Preliminary Bridge Design Calculations